Warwick School: Nathan PS 2 Tuesday, September 12, 2017.

For further references and links: http://dunfield.info/warwick2017

1. Put a hyperbolic structure
$$\Sigma = S^2 - \{\text{three points}\}$$
as follows. Fix an ideal triangle T in $\mathbb{H}^2$ and double it across the three boundary geodesics. That is, make two copies of it, call them T_1 and T_2, and glue the sides together by the "identity map". Prove that the hyperbolic metric on Σ is complete.

2. Suppose $\mathcal{T}$ is a (topological) ideal triangulation of punctured surface Σ. As per the lecture, we can put a hyperbolic structure on Σ by assigning a "shear" to each edge.

   (a) Formulate conditions on the shears that are equivalent to the hyperbolic structure being complete.

   (b) Use your answer in (a) and an Euler characteristic calculation to prove that the dimension of the Teichmüller space of a surface of genus g with k punctures has (real) dimension $6g - 6 + 2k$.

3. Consider the upper halfspace model for $\mathbb{H}^3$. The plane E at height 1 has the standard Euclidean metric; it is an example of a *horosphere*. Given a compact region R in E, consider the "chimney" over it:
$$C(R) = \left\{ (x, y, t) \in \mathbb{H}^3 \;\middle|\; (x, y, 1) \in R \text{ and } t \geq 1 \right\}$$
Prove that $C(R)$ has finite volume and relate its volume to the area of R.

4. Consider the a geodesic ideal tetrahedron T in $\mathbb{H}^3$. Recall from Lecture 2 that each edge of T has an associated *shape parameter*.

   (a) Prove that the shape parameter does not depend on an orientation of the edge and that opposite edges have the same shape parameter.

   Hint: Find symmetries of T by looking at the perpendicular bisectors between pairs of opposite edges.

   (b) Prove the formulas that relate the different edge parameters given in lecture.

5. Figure out how to get SnapPy to give you the edge equations of an ideal triangulation.