HW 11 for Math 191. Due Tuesday Dec. 17.

1. Let X and Y be independent random vars normally distributed with mean 0 and variance σ^2. Prove that $X+Y$ and $X-Y$ are independent.

2. Suppose that
$$F_X(t) = \frac{e^t}{1+e^t} \quad \text{for} \quad t \in \mathbb{R}.$$
Calculate the characteristic function, expectation, and variance of X.

3. A random variable X has a *lattice distribution* if for some $a, b \in \mathbb{R}$ then X is in $\{a + nb \mid n \in \mathbb{Z}\}$ with probability 1. Show that a var X has a lattice distribution if and only if $|\phi_X(t)| = 1$ for some $t \neq 0$.

4. Prove the weak law of large numbers using characteristic functions. (Note: The conclusion of the weak law of large numbers is *not* that S_n/n converges *in distribution* to a constant random variable.)

5. Let X_i be random vars, let P_i be the probability (measure) coming from X_i. We say that the X_i are tight if for every $\epsilon > 0$ there is a finite interval $(a, b]$ so that $P_i(a, b] > 1 - \epsilon$ for all i (equivalently, $F_{X_i}(b) - F_{X_i}(a) > 1 - \epsilon$).

   Suppose that X_i converge in distribution. Show that they are tight. Note: The converse is nearly true; if the X_i tight, then they have a subsequence that converges in distribution. This latter fact is one of the things you need to prove the continuity theorem.

6. Do (a) if you know complex analysis, and (b) if you don't

   (a) Let U be an open subset of $\mathbb{C}$. Let $f\colon U \to \mathbb{C}$ be a complex analytic (holomorphic) function. Show that if f' is nowhere 0 in U then f is conformal. Also, exhibit a non-trivial Riemann mapping from *some* U to $D = \{z \mid |z| < 1\}$.

   (b) Give an example of a conformal map other than the ones given in class.