Homework #6 Due Wed Mar 15

Reminder: There will be an in-class midterm next Friday (the 17th). More on this later.

Davenport: 4.13, 4.14, 5.02, 5.03

N1: Let $d, n \in \mathbb{N}$. Show that if $x^2 - dy^2 = n$ has one solution with $x, y \in \mathbb{N}$ then it has infinitely many.

N2: Let $a + b\sqrt{p}$ be the smallest solution to $x^2 - py^2 = 1$ there p is a prime congruent to 1 mod 4. Show that $x^2 - py^2 = -1$ has a solution with $x, y \in N$. If you like, you can follow the following outline:

- Show a is odd.
- Show $a \pm 1 = 2u^2$, $a \mp 1 = 2pv^2$, $2uv = b$.
- Show $u^2 - pv^2 = \pm 1$.
- Show that in the last step it's a -1.

N3: Show that the Diophantine equation $x^4 - 2y^4 = 1$ has no solutions with $x, y \in \mathbb{N}$.