

Math 416: //DRAFT ONLY// HW 9 due Friday, April 12, 2024.

Webpage: <http://dunfield.info/416>

Office hours: Wednesday 2:30–3:30pm and Thursday 2:00–3:00pm; other times possible by appointment. My office is 378 Altgeld.

Problems:

1. Section 5.3 of [FIS], Problem 1 parts (c–j).
2. Prove all of the following results:

Theorem. Let M be an $n \times n$ matrix having real nonnegative entries, let v be a column vector in \mathbb{R}^n having nonnegative coordinates, and let $u \in \mathbb{R}^n$ be the column vector all of whose entries are 1. Then:

- (a) M is a transition matrix if and only if $u^t M = u^t$.
- (b) v is a probability vector if and only if $u^t v = (1)$.

Corollary.

- (a) The product of two $n \times n$ transition matrices is also a transition matrix. Consequently, any power of a transition matrix is a transition matrix.
- (b) The product of a transition matrix and a probability vector is again a probability vector.

Note: This is Theorem 5.14 of the 5th edition of [FIS] and Theorem 5.15 of the 4th edition.

3. Prove that if a 1-dimensional subspace W of \mathbb{R}^n contains a nonzero vector with all nonnegative entries, then W contains a unique probability vector.
4. Section 5.3 of [FIS], Problem 7.
5. Section 6.1 of [FIS], Problem 3.
6. Section 6.1 of [FIS], Problem 8 parts (a) and (b).
7. Section 6.1 of [FIS], Problem 9.
8. Section 6.1 of [FIS], Problem 10 and Problem 15(a).