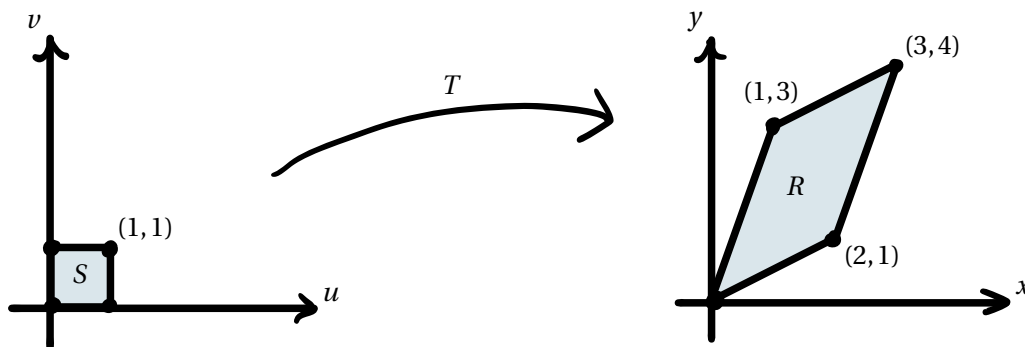


1. Consider the region R in \mathbb{R}^2 shown below at right. In this problem, you will do a change of coordinates to evaluate:

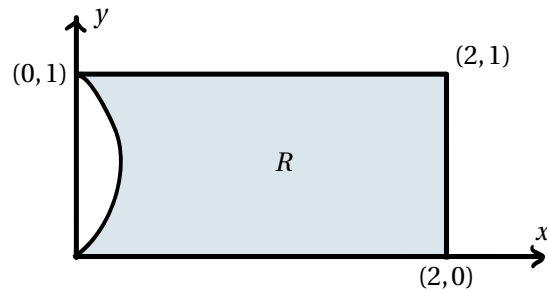
$$\iint_R x - 2y \, dA$$



- (a) Find a linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ which takes the unit square S to R . Write your answer both as a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and as $T(u, v) = (au + bv, cu + dv)$, and check your answer with the instructor.
- (b) Compute $\iint_R x - 2y \, dA$ by relating it to an integral over S and evaluating that. Check your answer with the instructor.
2. Another simple type of transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a translation, which has the general form $T(u, v) = (u + a, v + b)$ for a fixed a and b .
- (a) If T is a translation, what is its Jacobian matrix? How does it distort area?
- (b) Consider the region $S = \{u^2 + v^2 \leq 1\}$ in \mathbb{R}^2 with coordinates (u, v) , and the region $R = \{(x - 2)^2 + (y - 1)^2 \leq 1\}$ in \mathbb{R}^2 with coordinates (x, y) . Make separate sketches of S and R .
- (c) Find a translation T where $T(S) = R$.
- (d) Use T to reduce
- $$\iint_R x \, dA$$
- to an integral over S , and then evaluate that new integral using polar coordinates.
- (e) Check your answer in (d) with the instructor.

Problems 3 and 4 on the back.

3. Consider the region R shown below. Here the curved left side is given by $x = y - y^2$. In this problem, you will find a transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ which takes the unit square $S = [0, 1] \times [0, 1]$ to R .



- (a) As a warm up, find a transformation that takes S to the rectangle $[0, 2] \times [0, 1]$ which contains R .
- (b) Returning to the problem of finding T taking S to R , come up with formulas for $T(u, 0)$, $T(u, 1)$, $T(0, v)$, and $T(1, v)$. Hint: For three of these, use your answer in part (a).
- (c) Now extend your answer in (b) to the needed transformation T . Hint: Try “filling in” between $T(0, v)$ and $T(1, v)$ with a straight line.
- (d) Compute the area of R in two ways, once using T to change coordinates and once directly.
4. If you get this far, evaluate the integrals in Problems 1 and 2 directly, without doing a change of coordinates. It’s a fun-filled task . . .