

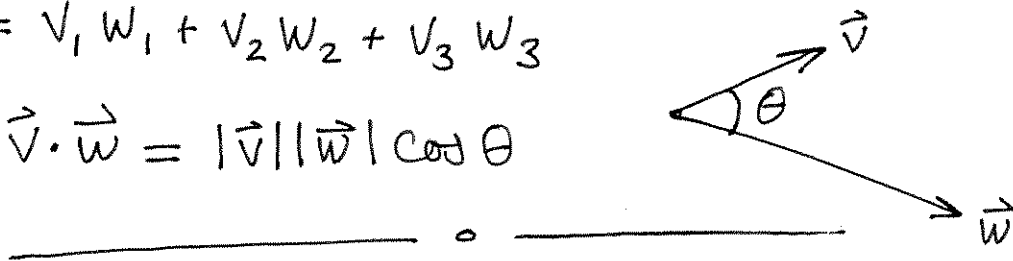
Lecture 3: Projection via the dot product (§12.3).

Lines and planes in \mathbb{R}^3 (§12.5).

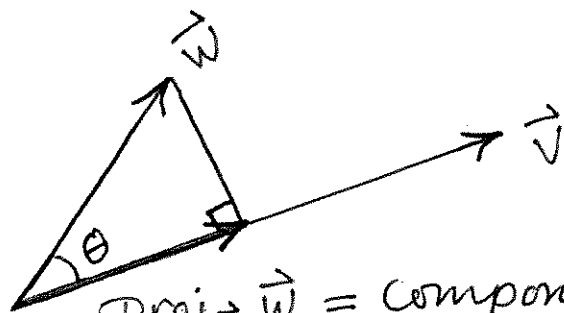
Last time: $\vec{v} = (v_1, v_2, v_3)$ $\vec{w} = (w_1, w_2, w_3)$

$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

Key: $\vec{v} \cdot \vec{w} = |\vec{v}| |\vec{w}| \cos \theta$



Projection:



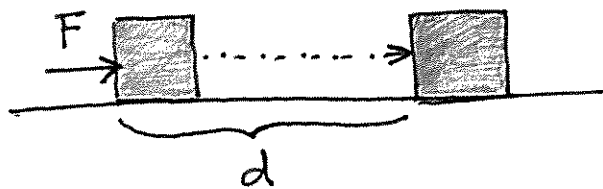
$\text{proj}_{\vec{v}} \vec{w}$ = component of \vec{w} along \vec{v} .
= scalar mult. of \vec{v}
closest to \vec{w} .

Note: $|\text{proj}_{\vec{v}} \vec{w}| = |\vec{w}| \cos \theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}|}$

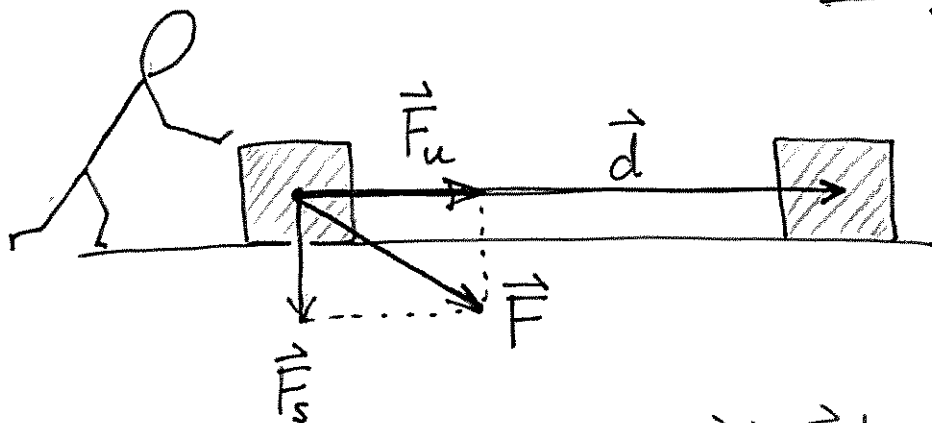
So if \vec{u} = unit vector pointing in same direction as \vec{v} = $\frac{\vec{v}}{|\vec{v}|}$, then

$$\begin{aligned} \text{proj}_{\vec{v}} \vec{w} &= |\vec{w}| \cos \theta \vec{u} = \left(\frac{\vec{v} \cdot \vec{w}}{|\vec{v}|} \right) \left(\frac{\vec{v}}{|\vec{v}|} \right) \\ &= \frac{\vec{v} \cdot \vec{w}}{|\vec{v}|^2} \vec{v} \end{aligned}$$

Work = (force) x (distance)



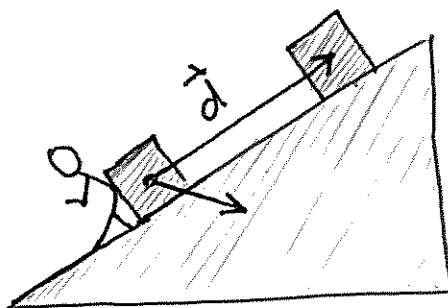
(2)



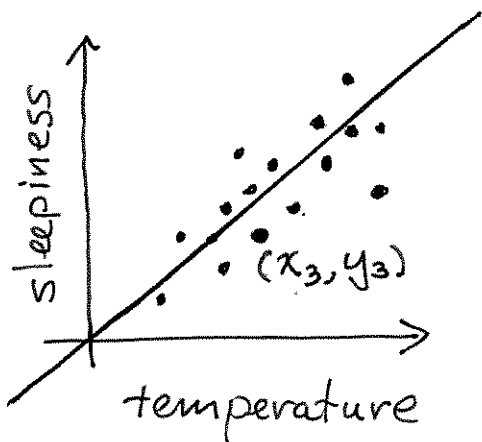
$$W = |\vec{F}_u| |\vec{d}| = |\text{proj}_{\vec{d}} \vec{F}| |\vec{d}| = \left(\frac{\vec{d} \cdot \vec{F}}{|\vec{d}|} \right) |\vec{d}|$$

$$= \vec{d} \cdot \vec{F}$$

So $W = \vec{F} \cdot \vec{d}$



Regression:



n samples (x_i, y_i)

Roughly, $y_i = c x_i$
What is c ?

Consider

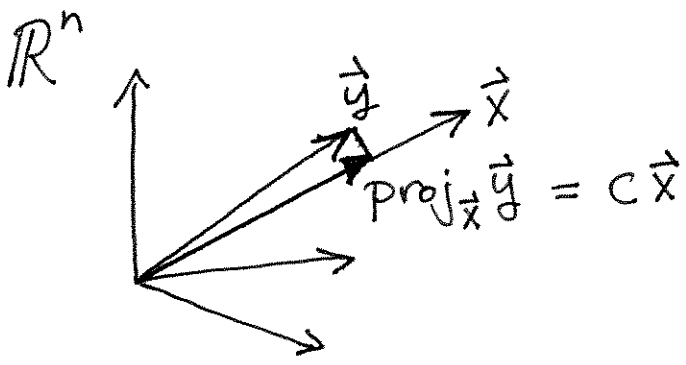
$$\vec{x} = (x_1, x_2, \dots, x_n) \text{ in } \mathbb{R}^n$$

$$\vec{y} = (y_1, y_2, \dots, y_n)$$

SKIP next time.

If $y_i = c x_i$ exactly for all i , then $\vec{y} = c \vec{x}$

(3)

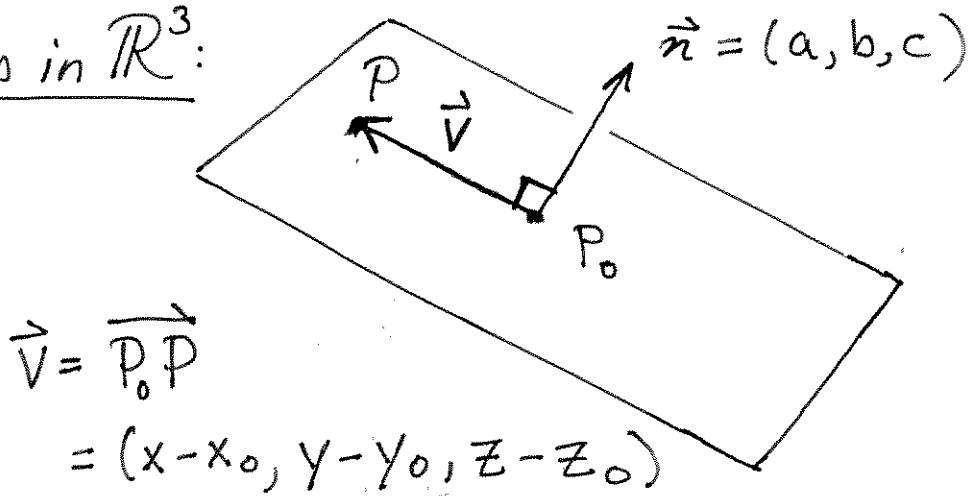


So, the best fit is given by

$$c = \frac{\vec{x} \cdot \vec{y}}{|\vec{x}|^2}$$

[In general, model has more parameters, and projection is onto a plane or a higher dimensional analog. See Math 415/416.]

Planes in \mathbb{R}^3 :



Can specify by a point $P_0 = (x_0, y_0, z_0)$ and a normal vector \vec{n} that meets the plane at right angles. Another point $P = (x, y, z)$ is in our plane exactly when \vec{n} and \vec{v} are perpendicular (orthogonal), i.e. $\vec{n} \cdot \vec{v} = 0$.

Thus P is in the plane when

(4)

$$\begin{aligned} 0 &= (a, b, c) \cdot (x - x_0, y - y_0, z - z_0) \\ &= a(x - x_0) + b(y - y_0) + c(z - z_0) \\ &= ax + by + cz + d \quad \text{where } d = -(ax_0 + by_0 + cz_0) \end{aligned}$$

Conversely,

$$ax + by + cz + d = 0$$

defines a plane, unless $a = b = c = 0$.

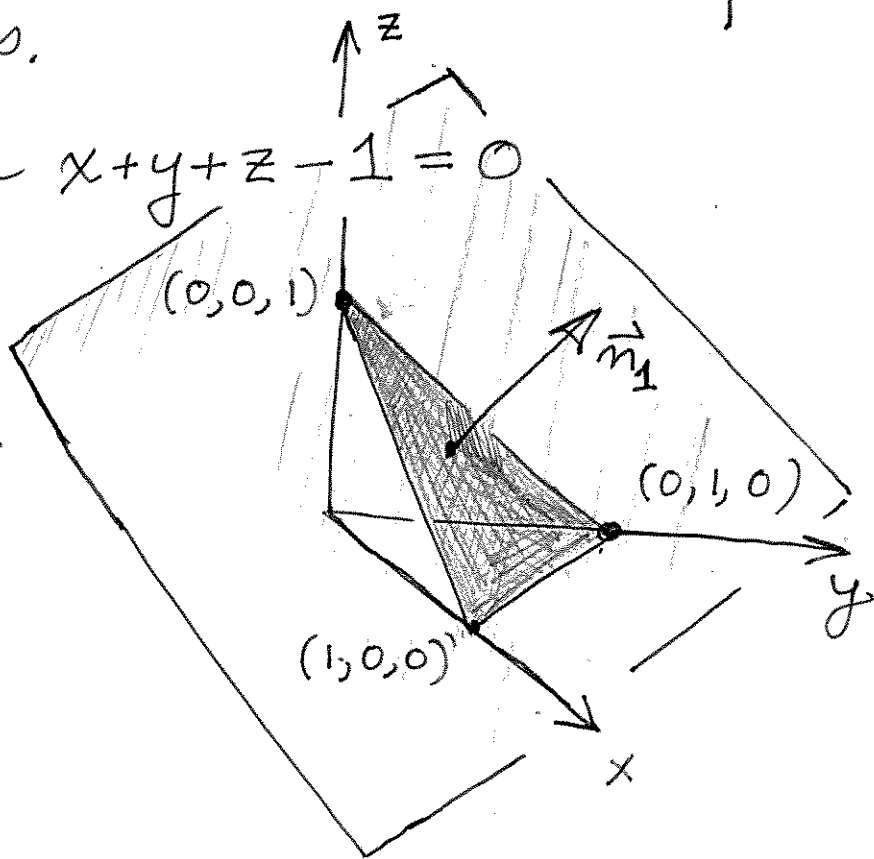
[Normal vectors will be key concept later in this course. For now, will use to solve geometric problems about planes.]

Ex: $P_1 =$ Plane given $x + y + z - 1 = 0$

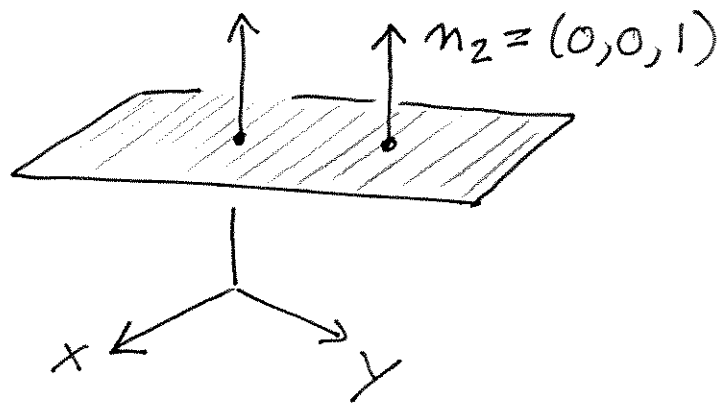
Any plane is determined by 3 pts.

Normal vector is

$$\vec{n}_1 = (1, 1, 1)$$



$$P_2 = \{z = 1\}$$

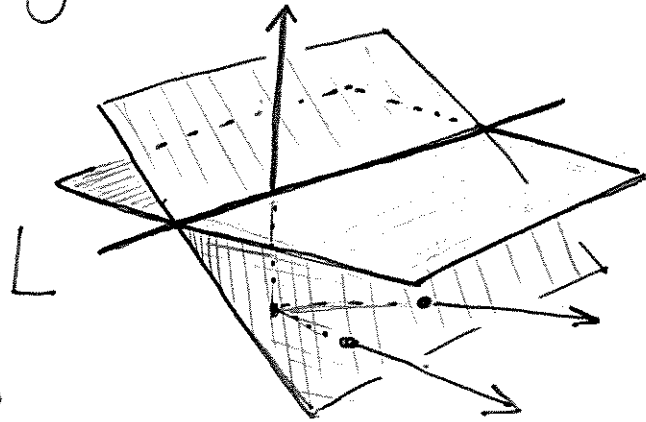


⑤

Q1: What is the intersection of P_1 and P_2

Q2: What is the angle between them?

A1: A line:



Points on L satisfy

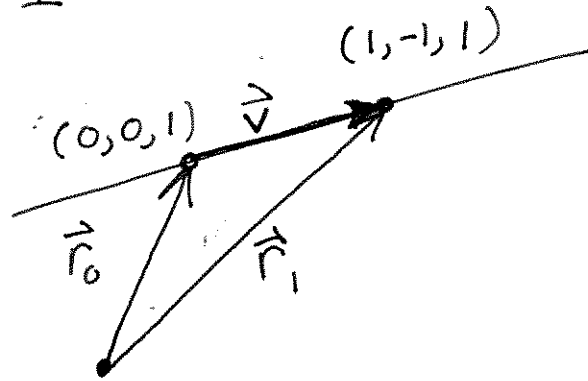
$$x + y + z = 1 \text{ and } z = 1$$

Two easy solutions:

$$\text{Get } \vec{v} = \vec{r}_1 - \vec{r}_0$$

$$= (1, -1, 1) - (0, 0, 1)$$

$$= (1, -1, 0)$$

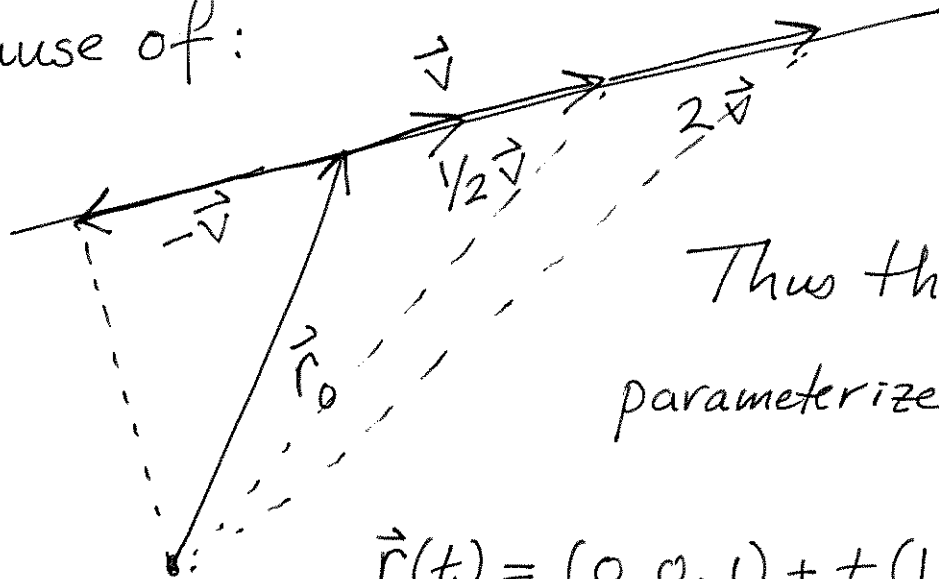


Thus every point on L has the form

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

⑥

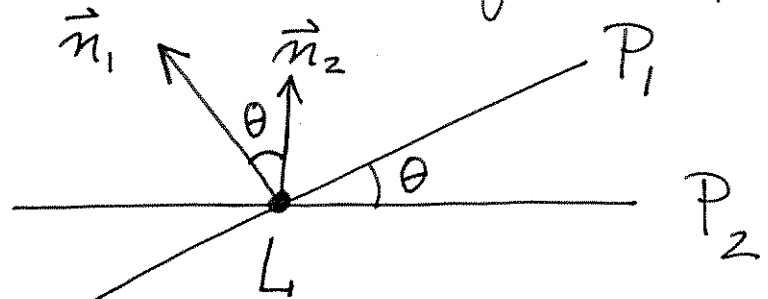
because of:



Thus the line L is parameterized by

$$\begin{aligned} \vec{r}(t) &= (0, 0, 1) + t(1, -1, 0) \\ &= (0, 0, 1) + (t, -t, 0) \\ &= (t, -t, 1) \end{aligned}$$

[This is just like Problem 3(e) from yesterday's worksheet.]



A2: Angle between the planes is the same as the angles between the normals.

$$\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} = \frac{1}{\sqrt{3} \cdot 1} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta \approx 54.7^\circ$$