

Math 416: Midterm the Third

Wednesday, April 20, 2016

Answer Key (answer)

Ex:0

There are 38 points possible on this exam. Take care to note that the problems are not weighted equally. Calculators, phones, books, notes (beyond the allowed one page) and suchlike aids to gracious living are not permitted. Show all your work as credit will not be given for correct answers without proper justification, except where indicated in Problem 6.

Do not start until instructed.

Good luck!



1. Suppose A and B are invertible matrices in $M_{n \times n}(\mathbb{R})$. Prove that $\det(ABA^{-1}B^{-1}) = 1$. **(5 points)** Ex:0

2. Consider the parallelepiped P in \mathbb{R}^3 determined by the three vectors $v_1 = (2, 0, 3)$, $v_2 = (0, 1, 2)$, and $v_3 = (0, 1, 0)$. Compute the *unsigned* volume of P . **(3 points)**



3. Let $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$.

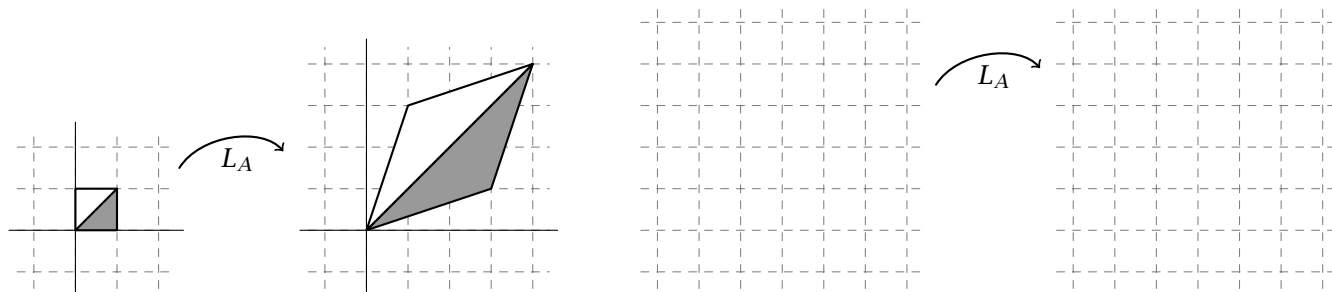
(a) Compute the characteristic polynomial of A . **(2 points)**

(b) Find all the eigenvalues of A . Hint: They are all integers. **(2 points)**

(c) For each eigenvalue compute the corresponding eigenspace. **(2 points)**

(d) Now diagonalize A , giving both a diagonal matrix D and an invertible matrix Q so that $D = Q^{-1}AQ$. **(3 points)**

(e) At left is our usual visualization of $L_A: \mathbb{R}^2 \rightarrow \mathbb{R}^2$. Use your answer in (a) to draw a more informative picture of L_A at right. **(1 point)**



4. A matrix $A \in M_{n \times n}(\mathbb{R})$ is called idempotent if $A^2 = A$. Prove that the only possible eigenvalues for an idempotent matrix A are 0 and 1. **(6 points)** Ex:0

5. Suppose the only the eigenvalues of $B \in M_{5 \times 5}(\mathbb{C})$ are 5 and 7 where $\dim(E_5) = \dim(E_7) = 2$. Determine whether B is diagonalizable and prove your answer. **(6 points)**



6. On these true/false and short answer questions you **do not need to justify your answers.**
(1 point each)

Ex:0

(a) A square matrix is diagonalizable if and only if its characteristic polynomial splits completely and it has distinct eigenvalues.

True False

(b) There exists a transition matrix where 2 is an eigenvalue.

True False

(c) Consider \mathbb{R}^3 with its usual inner product. If $v \in \mathbb{R}^3$ satisfies $\langle v, e_i \rangle = 0$ for $i = 1, 2, 3$ then $v = 0$.

True False

(d) The formula $\langle x, y \rangle = 2x_1y_1 + 3x_2y_2$ defines an inner product on \mathbb{R}^2 .

True False

(e) Give a matrix C in $M_{3 \times 3}(\mathbb{R})$ such that $C \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{pmatrix}$.

(f) Give a basis for $P_2(\mathbb{C})$ as a vector space over \mathbb{C} .

(g) Give a basis for $P_2(\mathbb{C})$ as a vector space over \mathbb{R} .

(h) The island of Madagascar is home to almost all of the worlds lemurs, a family of primitive non-ape primates. Divide Madagascar into 2 regions, the North and the South, along the Mania river. Suppose that in any given year, $1/3$ of the lemurs living in the North move to the South, and $1/6$ of those in the South move to the North. Give the transition matrix for the corresponding Markov chain.

