MATH 416. HW2.

#1 a). 3a. YES:

$$(-2 \ 0 \ 3) = \underline{-3}(2 \ 4 \ -1) + \underline{4}(1 \ 3 \ 0)$$

3c. No.

b). 4a: ~~NO~~ YES

$$x^3-3x+5 = \underline{3}(x^3+2x^2-x+1) \underline{-2}(x^3+3x^2-1)$$

4e: NO.

#2 a) 4g: $\begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} = \underline{3}\begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} + \underline{4}\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} + \underline{(-2)}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

YES.

4h: NO.

b). $\text{span}(S_1) = \{$all linear comb of vectors in $S_1\}$

$\forall v \in \text{span}(S_1)$, v is linear comb of vectors $v_i, i=1,\cdots,n$ in S_1

$\because S_1 \subseteq S_2 \quad \therefore v_i \in S_2$. $i=1,\cdots,n$.

$\therefore v$ is also a linear comb of vectors in S_2

$\therefore v \in \text{span}(S_2)$

$\therefore \text{span}(S_1) \subseteq \text{span}(S_2)$

c). $\because (0,1), (1,2) \in S$, & $(0,1), (1,2)$ are not parallel,

$\therefore \text{Span}\{(0,1),(1,2)\} = \mathbb{R}^2$

$\because \text{Span}\{(0,1),(1,2)\} \subseteq \text{Span}(S) \subseteq V = \mathbb{R}^2$

$\therefore \text{Span}(S) = \mathbb{R}^2$.

#3 Augmented matrix: $\left(\begin{array}{cc|c} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \\ 3 & 5 & 7 \end{array}\right)$

reduced echelon form: $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Solution: $(-1, 2)$

b) Augmented: $\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 1 & 1 & 2 & 0 \\ 3 & 8 & 1 & 1 \end{array}\right)$

reduced row echelon: $\begin{pmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Solution: ϕ.

c). Augmented: $\left(\begin{array}{cccc|c} 2 & 4 & 5 & 7 & 18 \\ 1 & 2 & 1 & -1 & 3 \\ 4 & 8 & 7 & 5 & 24 \end{array}\right)$

reduced row echelon: $\begin{pmatrix} 1 & 2 & 0 & -4 & -1 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Solution:

$(-1-2x+4y,\ x,\ 4-3y,\ y)$.

#4. a) $\because$ Multiplying row 1 of A by 1, we obtain A.

$\therefore$ $A \cong A$

c) Suppose by row operation $f_1, \cdots, f_n$, $A \xrightarrow{f_1 \cdots f_n} B$.

By row operation $g_1, \cdots, g_m$. $B \xrightarrow{g_1 \cdots g_m} C$

Then we have $A \xrightarrow{f_1 \cdots f_n \cdot g_1, \cdots g_m} C$

$\therefore$ A is row equivalent to C.

b) Suppose by row operation $f_1, \cdots, f_n$, $A \xrightarrow{f_1 \cdot f_2 \cdot \cdots \cdot f_n} B$.

Then we have $B \xrightarrow{f_n^{-1} \cdot f_{n-1}^{-1} \cdot \cdots \cdot f_1^{-1}} A$

#5 i) $\because A \cdot \bar{0} = 0 \quad \therefore \bar{0} \in \mathcal{N}(A)$

ii) ~~Suppose $B_1, B_2 \in \mathcal{N}(A)$~~

Suppose $v_1, v_2 \in \mathcal{N}(A)$, i.e. $Av_1 = 0$, $Av_2 = 0$.

Then $A(v_1 + v_2) = Av_1 + Av_2 = 0$

$\therefore v_1 + v_2 \in \mathcal{N}(A)$

iii) Suppose $v \in \mathcal{N}(A)$. i.e. $Av = 0$

Then $\forall a \in \mathbb{R}$ $A(av) = a \cdot (Av) = a \cdot 0 = 0$

$\therefore av \in \mathcal{N}(A)$

$\therefore \mathcal{N}(A)$ is a subspace.