Math 416: //DRAFT ONLY// HW 8 due Friday, November 4, 2022.

Webpage: http://dunfield.info/416

Office hours: Wednesday 2:30–3:30pm and Thursday 3:00–4:00pm; other times possible by appointment. My office is 378 Altgeld.

Problems:

1. Let $\mathbb{C}$ denote the field of complex numbers, as discussed in detail in Appendix D of [FIS]. As with any field, we can consider vector spaces, linear transformations, and matrices over $\mathbb{C}$ rather than over our usual field $\mathbb{R}$.

   (a) The complex numbers $\mathbb{C}$ can be viewed as a vector space over either $\mathbb{C}$ or $\mathbb{R}$ with the usual scalar multiplication. Prove that $\mathbb{C}$ has dimension 1 as a vector space over $\mathbb{C}$ but has dimension 2 as a vector space over $\mathbb{R}$. In each case, give an explicit basis.

   (b) Since $\mathbb{R}$ is a subset of $\mathbb{C}$, if V is a vector space over $\mathbb{C}$ then it is also a vector space over $\mathbb{R}$: just use the same scalar multiplication but restricted to scalars in $\mathbb{R}$. If V has dimension n as a vector space over $\mathbb{C}$, prove that it has dimension $2n$ as a vector space over $\mathbb{R}$. Hint: Use Theorem 2.19 from [FIS] to reduce to the case where V is just $\mathbb{C}^n$.

   (c) Diagonalize the following matrices over $\mathbb{C}$ by giving a $Q \in M_{2\times 2}(\mathbb{C})$ so that $Q^{-1}AQ$ is diagonal.

   $$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$$

2. Section 5.1 of [FIS], Problem 1.

3. Section 5.2 of [FIS], Problem 1.

4. Section 5.2 of [FIS], Problem 2, parts (e) and (g).

5. Section 5.2 of [FIS], Problem 3, parts (a) and (d).

6. Prove that similar matrices have the same characteristic polynomial.

7. Section 5.2 of [FIS], Problem 7.

8. If A is a square matrix prove that A and A^t have the same eigenvalues. Do they have the same eigenvectors? Either prove they do, or give a counterexample.

9. Suppose that A in $M_{n\times n}(\mathbb{R})$ has two distinct eigenvalues λ_1 and λ_2, and that $\dim(E_{\lambda_1}) = n-1$. Prove that A is diagonalizable.

10. Section 5.3 of [FIS], Problem 6. *5th edition changed wording and time period from months to weeks. Copy one version here.*