

**Math 416: //DRAFT ONLY// HW 7 due Friday, October 28, 2022.**

**Webpage:** <http://dunfield.info/416>

**Office hours:** Wednesday 2:30–3:30pm and Thursday 3:00–4:00pm; other times possible by appointment. My office is 378 Altgeld.

**Problems:**

1. Prove the following result that was used in class. Suppose  $E$  is the elementary matrix obtained from  $I_n$  by the row operation  $R$ , that is,  $I_n \xrightarrow{R} E$ . Prove that for all  $A \in M_{n \times n}(\mathbb{R})$  one has  $A \xrightarrow{R} EA$ . Said another way, left-multiplication by  $E$  implements the row operation that built  $E$  in the first place.
2. Prove that if  $A, B \in M_{n \times n}(\mathbb{R})$  are similar matrices then  $\det(A) = \det(B)$ .
3. A matrix  $Q \in M_{n \times n}(\mathbb{R})$  is called orthogonal if  $QQ^t = I_n$ .
  - (a) Prove that if  $Q$  is orthogonal then  $\det(Q) = \pm 1$ .
  - (b) Give examples of orthogonal matrices for  $n = 2$  with both possible values of the determinant.
4. Suppose  $A, B \in M_{n \times n}(\mathbb{R})$  satisfy  $AB = I_n$ .
  - (a) Use the determinant to prove that  $A$  is invertible.
  - (b) Prove or disprove:  $B = A^{-1}$ .
5. Section 5.1 of [FIS], Problem 2 parts (a) and (c).
6. Let  $T$  be a linear operator on a finite-dimensional vector space  $V$ .
  - (a) Show that  $T$  is invertible if and only if 0 is not an eigenvalue of  $T$ .
  - (b) If  $T$  is invertible, show that  $\lambda^{-1}$  is an eigenvalue of  $T^{-1}$  if and only if  $\lambda$  is an eigenvalue of  $T$ .
7. Suppose  $T: V \rightarrow V$  is a linear operator with  $V$  finite-dimensional. Suppose  $v \in V$  is an eigenvector of  $T$  with eigenvalue  $\lambda$ . As usual,  $T^m: V \rightarrow V$  denotes composition of  $T$  with itself  $m$  times. Prove that  $v$  is also an eigenvector for  $T^m$  and give a formula for the corresponding eigenvalue.
8. Section 5.1 of [FIS]. Problem 4(a) if using the 5th edition, but Problem 3(a) if using the 4th.
9. Section 5.1 of [FIS], Problem 5 parts (b) and (h) if using the 5th edition, but Problem 4 parts (b) and (h) if using the 4th.