Math 416: HW 6 due Friday, October 14, 2022.

Webpage: http://dunfield.info/416

Office hours: Wednesday 2:30–3:30pm and Thursday 3:00–4:00pm; other times possible by appointment. My office is 378 Altgeld.

Problems:

1. Suppose \( T: V \to W \) is a linear transformation between finite-dimensional vector spaces, and let \( \beta = \{v_1, \ldots, v_n\} \) be a basis for \( V \). Prove that \( T \) is an isomorphism if and only if \( \gamma = \{w_1, \ldots, w_n\} \) where \( w_i = T(v_i) \) is a basis for \( W \).

   Hint: We did part of this in class.

2. Section 2.5 of [FIS], Problem 1.

3. Section 2.5 of [FIS], Problem 2 (a–c) and Problem 3 (c) and (d).

4. Section 2.5 of [FIS], Problem 6 (a) and (c).

5. Section 2.5 of [FIS], Problem 7.

6. Compute the determinants of the following matrices:

   (a) \[
   \begin{pmatrix}
   1 & 2 \\
   3 & 4
   \end{pmatrix}
   \]

   (b) \[
   \begin{pmatrix}
   -4 & 2 \\
   3 & -4
   \end{pmatrix}
   \]

   (c) \[
   \begin{pmatrix}
   2 & 3 \\
   -2 & -3
   \end{pmatrix}
   \]

7. Suppose \( A \in M_{2\times2}(\mathbb{R}) \).

   (a) Show that \( \det(A) = \det(A^t) \).

   (b) Show that if \( B \) is obtained from \( A \) by swapping the two rows, then \( \det(B) = -\det(A) \).

   (c) How does the determinant change if instead you swap the columns of \( A \)?

   (d) If \( B \) is also in \( M_{2\times2}(\mathbb{R}) \), prove that \( \det(AB) = \det(A) \det(B) \).

8. Section 4.1 of [FIS], Problem 10.

9. Section 4.2 of [FIS], Problems 5 and 11.

10. Section 4.2 of [FIS], Problem 21.