

## Math 416: HW 5 due Friday, October 7, 2022.

Webpage: <http://dunfield.info/416>

Office hours: Wednesday 2:30–3:30pm and Thursday 3:00–4:00pm; other times possible by appointment. My office is 378 Altgeld.

Textbook: In the assignment, the main text is abbreviated as follows:

[FIS] Freidberg, Insel, Spence, *Linear Algebra*, 4th or 5th edition, 2002 or 2019.

### Problems:

1. Section 2.3 of [FIS], Problem 1.
2. Section 2.3 of [FIS], Problem 2.
3. Give matrices  $A, B \in M_{2 \times 2}(\mathbb{R})$  where  $AB$  is the zero matrix but  $BA$  is not.
4. Section 2.3 of [FIS], Problem 3.
5. Section 2.3 of [FIS], Problem 4 (a, b).
6. Section 2.4 of [FIS], Problem 1.
7. Suppose  $A$  and  $B$  are invertible  $n \times n$  matrices.
  - (a) Prove that  $(AB)^{-1} = B^{-1}A^{-1}$ .
  - (b) Prove that  $(A^t)^{-1} = (A^{-1})^t$
8. Let  $V$  be a finite-dimensional vector space over  $\mathbb{R}$  with basis  $\beta$ , and set  $n = \dim V$ . Consider the linear transformation  $\phi_\beta: V \rightarrow \mathbb{R}^n$  defined by  $\phi_\beta(v) = [v]_\beta$ .
  - (a) Prove that  $\phi_\beta$  is an isomorphism.
  - (b) Use part (a) to show that any two vector spaces of dimension  $n$  are isomorphic. This gives an alternate proof of Theorem 2.19 of [FIS].
9.
  - (a) Let  $A$  and  $B$  be  $n \times n$  matrices such that  $AB$  is invertible. Prove that both  $A$  and  $B$  are invertible.
  - (b) Give an example of two noninvertible matrices whose product is invertible. Hint: Look at non-square matrices.
  - (c) Prove or give a counterexample: If  $A$  and  $B$  are nonzero  $n \times n$  matrices with  $AB$  the zero matrix then  $A$  is not invertible.
10. Find the inverse of the following matrix, and give a direct check of your answer.

$$A = \begin{pmatrix} 4 & -2 & -5 \\ -4 & 1 & 4 \\ -3 & 1 & 3 \end{pmatrix}$$