Math 416: HW 4 due Friday, September 30, 2022.

Webpage: http://dunfield.info/416

Office hours: Wednesday 2:30–3:30pm and Thursday 3:00–4:00pm; other times possible by appointment. My office is 378 Altgeld.

Textbooks: In the assignment, the main text is abbreviated as follows:

[FIS] Freidberg, Insel, Spence, *Linear Algebra*, 4th or 5th edition, 2002 or 2019.

Problems:

1. Section 2.1 of [FIS], Problem 1.
2. Section 2.1 of [FIS], Problems 2 and 3.
3. Section 2.1 of [FIS], Problem 9 (a, b, c).
4. Section 2.1 of [FIS], Problem 10.
5. Section 2.1 of [FIS], Problems 18.
6. Let V, W be vector spaces, with $\dim(V) = n$, $\dim(W) = m$, and $n > m$.
   (a) Show that there is no one-to-one linear transformation $T\colon V \to W$.
   (b) Show that there is no onto linear transformation $T\colon W \to V$ (notice that V, W have flipped in this expression!)
   (c) Show that a linear map $T\colon V \to W$ need not be onto by giving an example where it is not.

   Hint: See Appendix B of [FIS] for the definitions of "onto" and "one-to-one" and consult Theorems 2.4 and 2.5 in §2.1 of [FIS].
7. We define the linear transformation $T_\theta\colon \mathbb{R}^2 \to \mathbb{R}^2$ to be rotation counter-clockwise about the origin through angle θ. Let T_x be the transformation that reflects in the x-axis.
   (a) Write down the matrices of T_θ and T_x with the respect to the standard basis $\beta = \{e_1, e_2\}$ for $\mathbb{R}^2$.
   (b) Show that for $\theta \in (0, \pi) \cup (\pi, 2\pi)$ one has
   $$T_x \circ T_\theta \neq T_\theta \circ T_x.$$
   (c) Next, show that there is some angle ψ such that
   $$T_x \circ T_\psi = T_\theta \circ T_x.$$
   What is the relationship between θ and ψ? Discuss the geometric meaning of this computation.
8. Section 2.2 of [FIS], Problem 2 (a, b, c).
9. Section 2.2 of [FIS], Problem 3.
10. Section 2.2 of [FIS], Problem 5.