
Webpage: http://dunfield.info/416

Office hours: Wednesday 2:30–3:30pm and Thursday 3:00–4:00pm; other times possible by appointment. My office is 378 Altgeld.

Textbooks: In the assignment, the two texts are abbreviated as follows:


Problems:

1. In these questions, you will determine whether one vector is a linear combination of two others.
   (a) Section 1.4 of [FIS], Problem 3: parts (a) and (c).
   (b) Section 1.4 of [FIS], Problem 4: parts (a) and (e).

2. In class, I defined the span of a finite list of vectors $u_1, u_2, \ldots, u_n$. More generally, given a nonempty subset $S$ of a vector space $V$, one defines span$(S)$ to be the set of all linear combinations of vectors in $S$. Here are some problems about the span.
   (a) Section 1.4 of [FIS], Problem 5: parts (g) and (h).
   (b) Suppose $S_1$ and $S_2$ are subsets of a vector space $V$. Show that if $S_1$ is contained in $S_2$, then span$(S_1)$ is contained in span$(S_2)$.
   (c) Let $V = \mathbb{R}^2$ and $S = \{(x, y) \text{ where } x \geq 0 \text{ and } y \geq x\}$. Find span$(S)$.

3. Solve each of the following linear systems by writing down its augmented matrix, doing row operations to get a matrix in reduced row echelon form, and using that to find all of the solutions. You should label your row operations as in §RREF of [B].
   (a)
   \[
   \begin{align*}
   2x_1 + x_2 &= 0 \\
   x_1 + x_2 &= 1 \\
   3x_1 + 4x_2 &= 5 \\
   3x_1 + 5x_2 &= 7
   \end{align*}
   \]
(b) 

\[ \begin{align*} 
  y_1 + 2y_2 - y_3 &= 1 \\
  y_1 + y_2 + 2y_3 &= 0 \\
  5y_1 + 8y_2 + y_3 &= 1 
\end{align*} \]

(c) 

\[ \begin{align*} 
  2x_1 + 4x_2 + 5x_3 + 7x_4 &= 18 \\
  x_1 + 2x_2 + x_3 - x_4 &= 3 \\
  4x_1 + 8x_2 + 7x_3 + 5x_4 &= 24 
\end{align*} \]

4. Suppose that \( A, B, \) and \( C, \) are \( m \times n \) matrices with real coefficients. Prove the following three facts from the definition of row equivalence.

(a) \( A \) is row equivalent to \( A. \)

(b) If \( A \) is row equivalent to \( B, \) then \( B \) is row equivalent to \( A. \)

(c) If \( A \) is row equivalent to \( B, \) and \( B \) is row equivalent to \( C, \) then \( A \) is row equivalent to \( C. \)

Note: A relationship that satisfies these three properties is known as an equivalence relation; this is a formal way of saying that a relationship behaves like equality, without requiring the relationship to be as strict as equality itself.

5. Suppose \( A \) is an \( m \times n \) matrix with real entries. The null space of \( A, \) denoted \( \mathcal{N}(A), \) is the set of all solutions in \( \mathbb{R}^n \) to the linear system \( LS(A,0), \) where here \( 0 \) is the zero vector in \( \mathbb{R}^m. \) Prove that \( \mathcal{N}(A) \) is a subspace of \( \mathbb{R}^n. \)