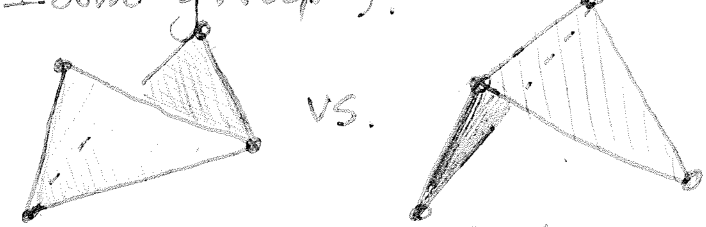
Lecture 17: Applications of verified hyp strs.

[Finish Demo?]

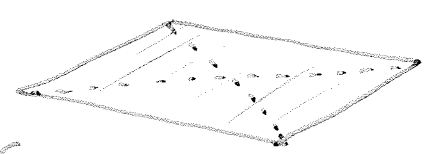
Setting: $[z]$ in $(\mathbb{I}\mathbb{C})^n$ which, by Krawczyk, contains a sol'n to edge and cusp eqns for $\mathcal{T}$ with each $\mathrm{Im}(z_i) > 0$.

Applications: Rigorous methods for

A) Finding canon. cellulation ($\Rightarrow$ solve homeo prob, commute Isom group).



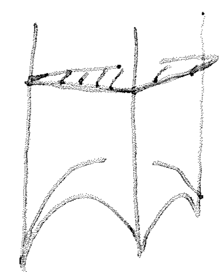
Works well when it is a triangulation [Hoffmann-Licata-D]. But more general cells are a problem since equality in $\mathbb{I}\mathbb{R}$ is gen. meaningless.



Fallback: exact sol'ns in some # field.

B) Find all non hyperbolic Dehn fillings (assume one cusp).

<u>Point</u>: have very accurate desc. of T, a cross-sec. horotorus, so can find all slopes



where $\dfrac{\operatorname{len}(\alpha)}{\sqrt{\operatorname{area}(T)}} \leq 7.6$

To get a finite set of slopes to examine.

With T embedded, can also apply

[Gromov-Thurston] If $\operatorname{len}(\alpha) \geq 2\pi$ then $M(\alpha)$ has a metric of neg. curve $\Rightarrow$ hyp.

[Agol-Lackenby] If $\operatorname{len}(\alpha) \geq 6$ then same.

C) Solve the word problem in $\pi_1 M$.

G a gp gen by $s_1, \ldots, s_n$

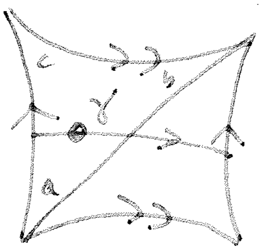
w a word in s_i

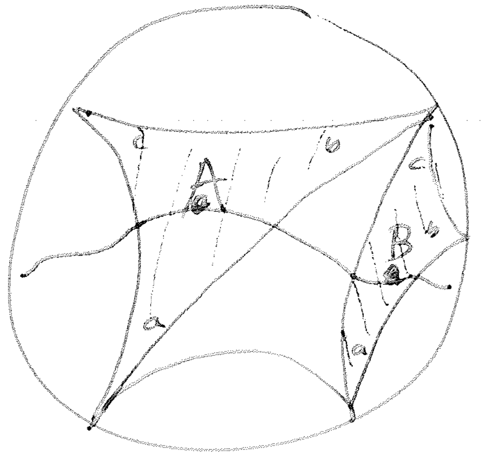
<u>Q</u>: Is $w = 1$ in G?

no kernel!

<u>Recall</u>: $M = \mathbb{H}^3 / \rho(\pi_1 M)$ where $\rho : \pi_1 M \to \operatorname{Isom}^+(\mathbb{H}^3)$ is the holonomy rep'n. [How do find hol reps?]

Compute via the developing map:





$\rho(\gamma)$ takes A to B.

<u>So</u>: If $[Z]$ contains the hyp str for J

then get $\rho : \pi_1 M \longrightarrow PSL_2(\mathbb{I}\mathbb{C})$ ← det $\not\ni 0$ not clsd under mult.

with caveats: ↖ really $\mathrm{FreeGrp}(S = (s_1, s_2, \ldots, s_n))$

Given $w \in \mathrm{FreeGrp}(S)$ look at $\rho(w)$.

If $I \notin \rho(w)$ then $w \neq 1$ in $\pi_1 M$.

<u>Thm</u> [D] Suppose $tr(\rho([s_1, s_2]))$ does not contain 2.
If w is a word in S with

$$|tr^2(\rho(w)) - 4| + |tr([\rho(s_i), \rho(w)]) - 2| < 1$$

for $i = 1, 2$, then $w = 1$ in $\pi_1 M$.

Easy cor of Jørgensen's inequality: If A, B in $PSL_2\mathbb{C}$ generate a discrete <u>nonelementary</u> subgp, then

$$|\mathrm{tr}^2(A) - 4| + |\mathrm{tr}([A,B]) - 2| \geq 1.$$

<u>nonelementary</u>: limit set is at most 2 pts.

If Γ is discrete, torsion free, the element gps $(\neq 1)$ are

a) cyclic gp of parabolics $\Gamma = \langle z \mapsto z+1 \rangle$

b) $\mathbb{Z}^2$ gp of parab $\Gamma = \left\langle \begin{matrix} z \mapsto z+1 \\ z \mapsto z+\tau \end{matrix} \right\rangle$

c) cyclic gp of hyp $\Gamma = \langle z \mapsto \lambda z \rangle$ $\lambda \in \mathbb{C} \setminus \{0,1\}$.

<u>Moral</u>: For disc. nonelem gp, if A is close to I then B is not and neither is $[A,B]$.