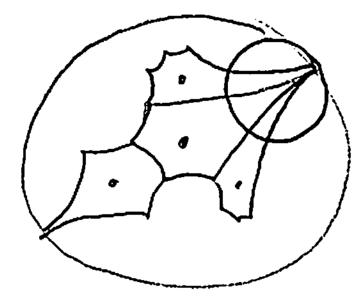
Lecture 11: More on closed hyp. 3-mflds

Thm: M^3 finite-vol hyp. Given $L > 0$, there are finitely many clsd geod of len $< L$.

Pf when have cusps: Dirichlet domain D is now noncpt. Fix cusp nhds C_i in M. Then $P = D \setminus \bigcup \tilde{C}_i$ is cpt and every clsd geod g has a lift $\tilde{g}$ meeting P.

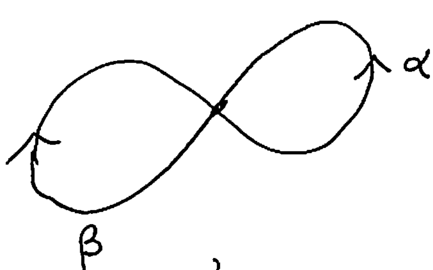


[Reason: no clsd geod in a C_i] ▨

Lemma: When M^3 is clsd, every systole is simple.

i.e. an emb. S^1

Pf: If the systole has a self-intersection then at least one of $\alpha, \beta \neq 1$, say α. Then α cor to a clsd geod h with $\text{len}(h) < \text{len}(\alpha) < \text{len}(\gamma)$, a contradiction. ▨



$\gamma = \alpha * \beta$

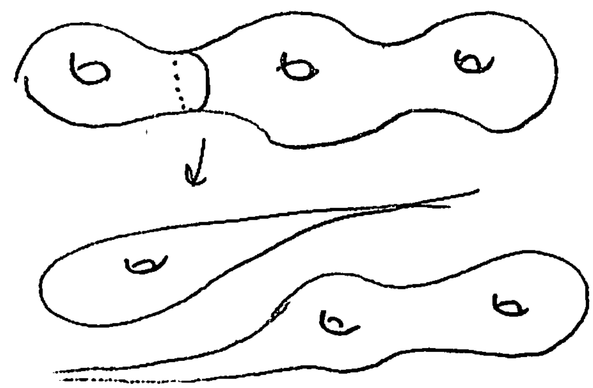
[Why does this arg. fail when have cusps?]

Fun facts: [Chinburg-Reid] $\exists$ clsd hyp M^3 where every closed geod is simple.

[Adams-Hass-Scott] Every finite-vol hyp M^3 has a simple geodesic.

<u>Thm</u>: Let g be any simple clsd geod in a clsd hyp M^3. Then $M \setminus g$ has a <u>complete</u> hyp. metric of finite vol.

<u>Note</u>: Also holds for surfaces:



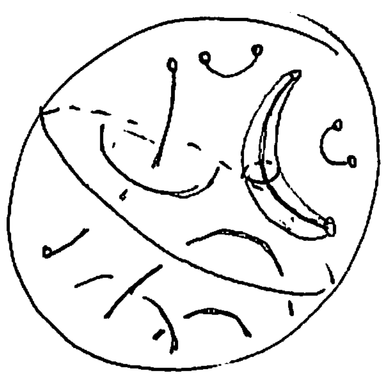
<u>Pf idea</u>: Check that $N = M \setminus \mathring{N}(g)$ is

a) irreducible

b) atoroidal (only incomp torus is ∂N).

c) not Seifert fibered

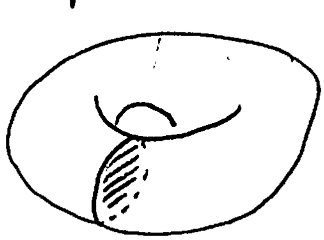
and then apply Geometrization [For Haken mflds]



a) Follows as M is irred and no clsd geod is contained in a ball.

c) Otherwise M is itself Seifert fib or reducible.

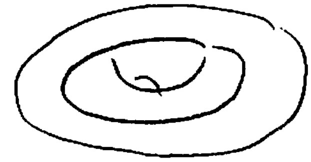
b) Suppose T is an incomp torus in N. As T compresses in M, either i) T bounds a solid torus $S = D^2 \times S^1$ in M or



ii) $T \subseteq$ ball in M.

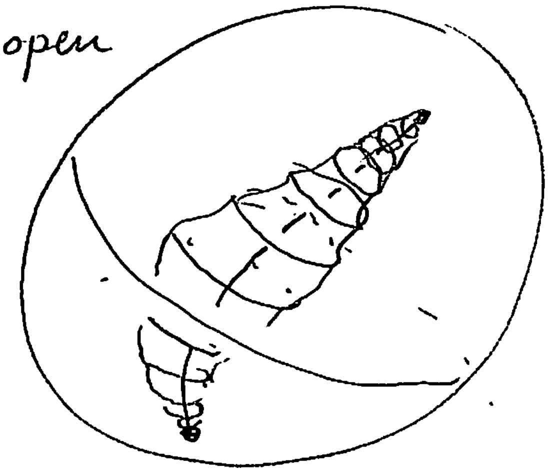
i) Will show g is a core curve for S

Let $\gamma \in \pi_1 M$ rep. the core of



S. Then $\pi_1 T \to \pi_1 M$ has image $\langle \gamma \rangle$

so $\tilde{T} \subseteq \mathbb{H}^3$ is a γ-invariant open annulus, limiting on the f.p. of γ. The only geod "inside" $\tilde{T}$ is the one joining those f.p.



ii) T lifts to $\mathbb{H}^3$

Then $\tilde{g}$ passes through the

knotted hole, which is impossible as $\tilde{g} \subseteq \bar{D}^3$ is unknotted. $\blacksquare$

Solving Homeo Prob for clsd hyp A, B

1) Find all systoles for A, B.

2) Find all Isom $(A \setminus \text{sys})$ to $(B \setminus \text{sys})$

3) See if any in ② extend to A, B.

Point in 3: X with $X \setminus \partial X = A \setminus sys_A$

Y with $Y \setminus \partial Y = B \setminus sys_B$

Then do Dehn filling: (attach solid tori)

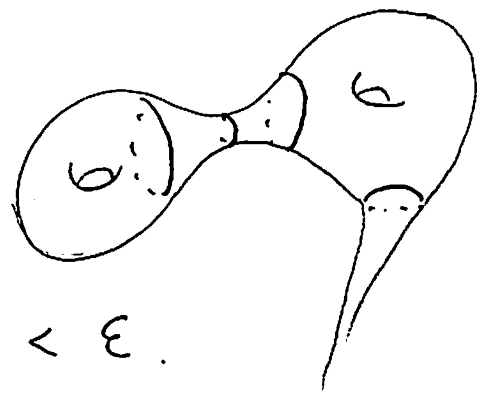
$A = X(\alpha) \quad B = Y(\beta)$ for s.c.c. $\alpha \subseteq \partial X$, $\beta \subseteq \partial Y$.

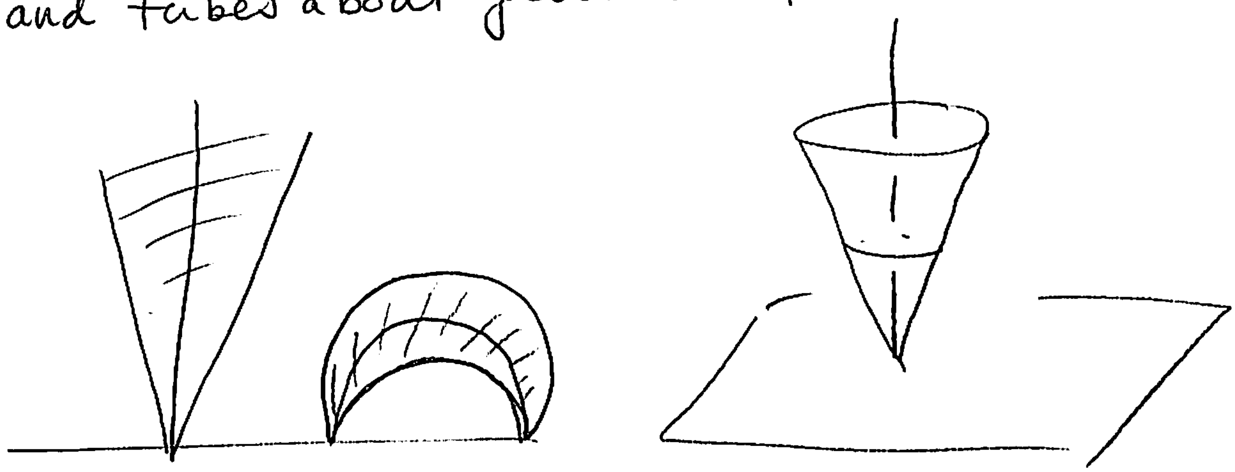
So using that there are finitely many Bom in 2.

Margulis Lemma: $\exists \varepsilon_n > 0$ such for all finite vol hyp n-mflds M the set

$$M_{thin,\varepsilon} = \{ x \in M \mid inj_x M < \varepsilon \}$$

is a finite union of cusps nbhds and tubes about geod γ of len $< \varepsilon$.





[Show inside view]