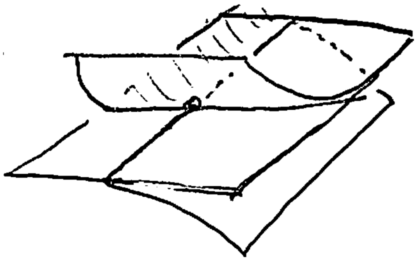
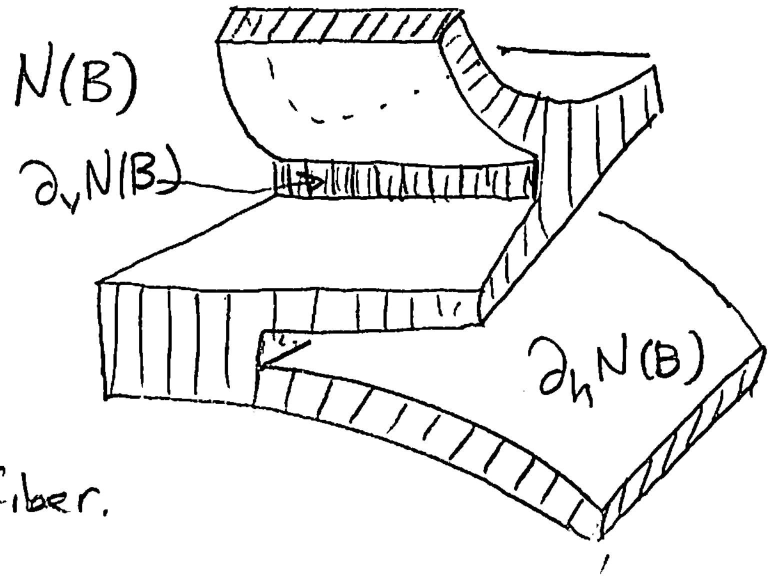
Lecture 23: Triangulations to taut foliations

Last time:

Branched surface B





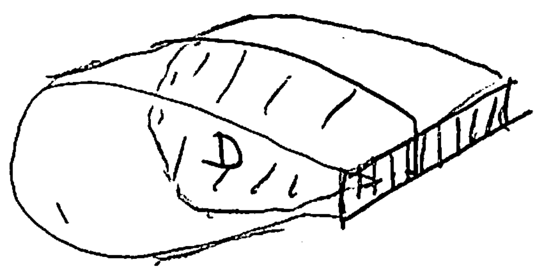
B carries S, Λ: iso into $N(B)$ so transv to I-fibers

fully carries: meets ever I-fiber.

B is laminar when

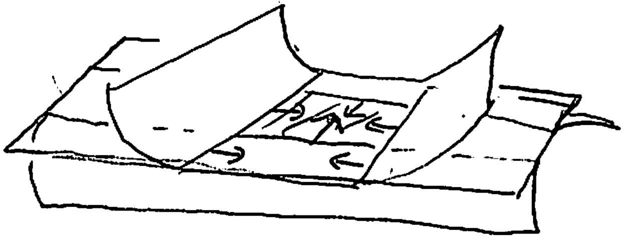
1) $\partial_h N(B)$ is incomp in $M \setminus \mathring{N}(B)$ (manifold w/ a collection of annuli in its ∂), no comp of $\partial_h N(B)$ is a ☺, $M \setminus \mathring{N}(B)$ is irreducible

2) No monogons $M \setminus \mathring{N}(B)$: A disc D is a comp R of $M \setminus \mathring{N}(B)$ with $R \cap D = \partial D$ and $D \cap \partial_v R$ is a single I-fiber



3) B does not carry a torus bdng a solid torus.

4) No sink discs, i.e a comp of $B \setminus$ (sing locus) where the maw v.f. pts in every where.



[Li] Let M^3 be clsd and orient.

a) Any laminar branched surface fully carries an ess. lamination.

b) Any ess. lamination Λ which is nowhere dense and where some leaf is not a plane is fully carried by a laminar branchd surface.

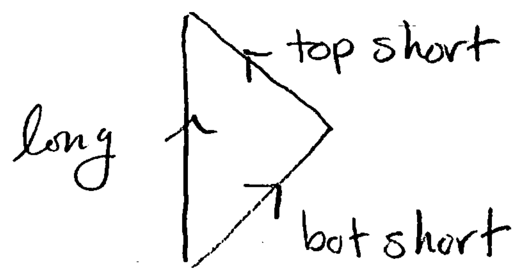
Note: If all leaves of Λ are planes in b then $M = T^3$ and Λ obtained from a fol. by irrat'l planes.

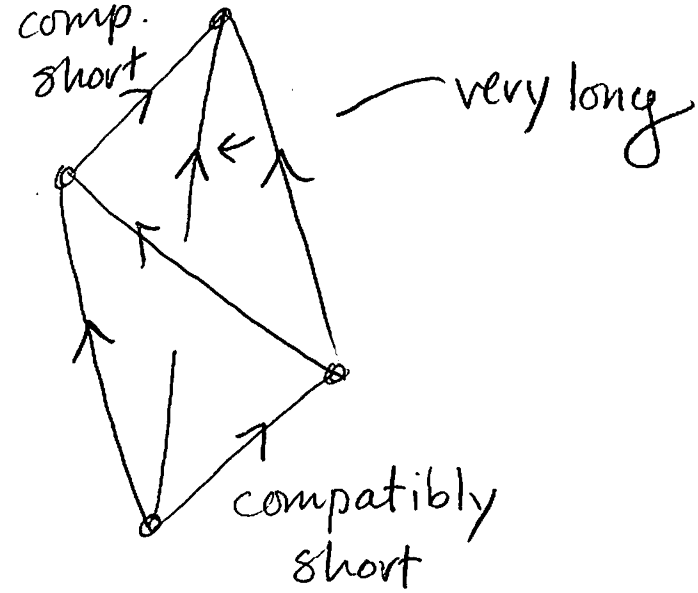
[Won't be able to say anything about the proof...]

$\mathcal{T}$ triangulation of clsd orient M^3
= collection of tets with faces glued in pairs.

An edge orient of $\mathcal{T}$ is a choice u of direction on each edge in $\mathcal{T}^{(1)}$. u is acyclic if no face of $\mathcal{T}^{(2)}$ is a direct. cycle:

Acyclic edge orient:





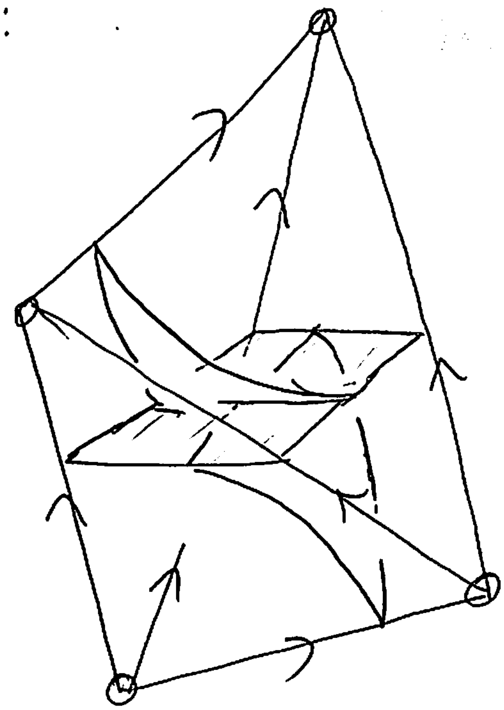
A sink edge in $\mathcal{T}^{(1)}$ is one that is very long in every tet where it appears

face relation on $\mathcal{T}^{(2)}$ is the equiv rel gen by the rule that two faces on the same tet that share a comp. short edge are equiv.

foliar orient: an acyclic orient with no sink edges and where the face relation has a single equiv. class.

Thm: Suppose $\mathcal{T}$ has one vertex and a foliar orient o. Then M has a co-orient taut fol $\mathcal{F}$ transv. to $\mathcal{T}^{(1)}$ inducing the edge orient o.

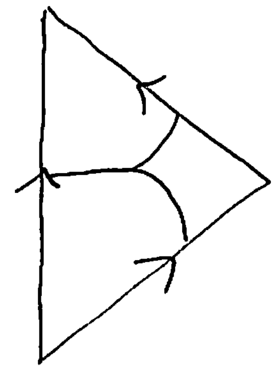
Proof idea:



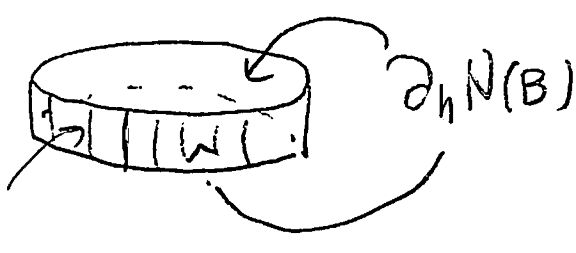
B(u)

a co-orient branched surface.

Fits together OK since



Now $R = M \setminus N(B)$ is a 3-ball, a nbhd of the single vertex in $\mathcal{T}^{(0)}$. Face rel having one equiv class $\Rightarrow$ $\partial_v N(B)$ is a single annulus

$\Rightarrow$ $R =$ 

Check cond: ① and ② : clear.

③ If B carries a torus T, let e be an edge of $\mathcal{T}^{(1)}$ that meets T. Then $T \cap e > 0$ so T is non-separating as e is a loop.

(4) Only one sector in each tet that <u>could</u> be a sink disc: the one meeting the very long. edge So: no sink edge $\Rightarrow$ no sink disc.

So $B(u)$ carries an ess. lam. Λ. Now $N(B(u))$ is an orient I-bundle, so

$N(B(u)) \setminus\setminus \Lambda$ is a product $\Longrightarrow$

$M \setminus\setminus \Lambda$ is $\underbrace{\text{a product}}_{\text{over pos. a noncpt surface}}$ $\Longrightarrow$ fill in to a fol $\mathcal{F}$.

Finally $\mathcal{F}$ is taut since the edges of $\mathcal{T}^{(1)}$ give clsd transv. collectively meeting every leaf. $\blacksquare$

Conj M^3 clsd irred. M has a co-orient taut fol $\Leftrightarrow$ M is not an L-space ($\widehat{HF}_{red}(M) \neq 0$).

Thm: Of some coll $\mathcal{Y}$ of 307,301 hyp. M^3 with $b_1 = 0$.

a) 47% are L-spaces 53% are not.

b) at least 162,341 admit taut fol coming from foliar orient. (99.6% of all non L-spaces)

c) At least 26.1% have orderable π_1

d) At least 36.1% have nonord π_1

In ⓒ, 10% (of total) come from actions on S^1_{univ} with $e = 0$. Cf

[Boyer-Hu] If $\mathcal{F}$ is a co-orient taut fol of an atoroidal M, then $e(\rho_{univ}) = e(T\mathcal{F})$.