Lecture 21: Essential Laminations

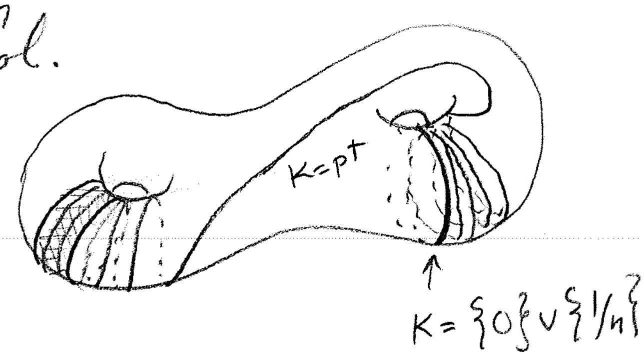
So far: taut fol and incomp. surf.

Def: A (surface) lamination Λ of M^3 is a foliation of a closed subset C of M. That is, a decomp of C into connected surfaces (leaves) with foliated charts $\mathbb{R}^2 \times \mathbb{R}$ which C meets as $\mathbb{R}^2 \times K$ for $K \subseteq \mathbb{R}$ either cpt or all of $\mathbb{R}$

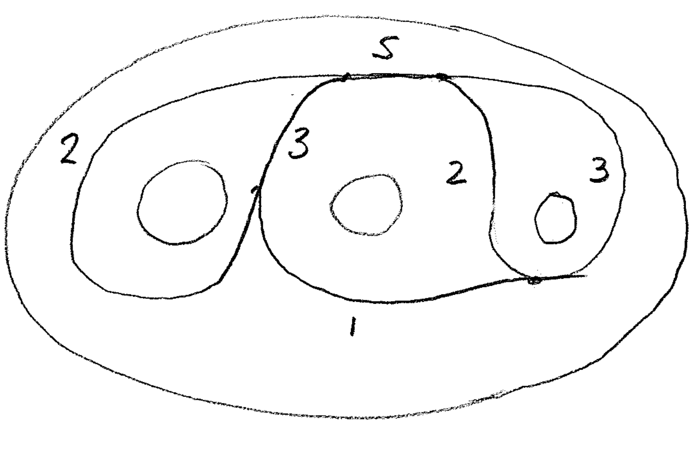
Exs: 1) Fol 2) surface ($K = \{pt\}$) 3) surface $\times [0,1]$.

4) $C = \overline{L}$ where L is a leaf a fol.

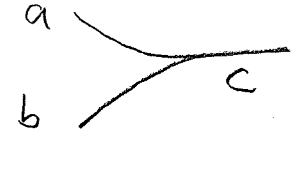
5) K = Cantor set. will be the most common.



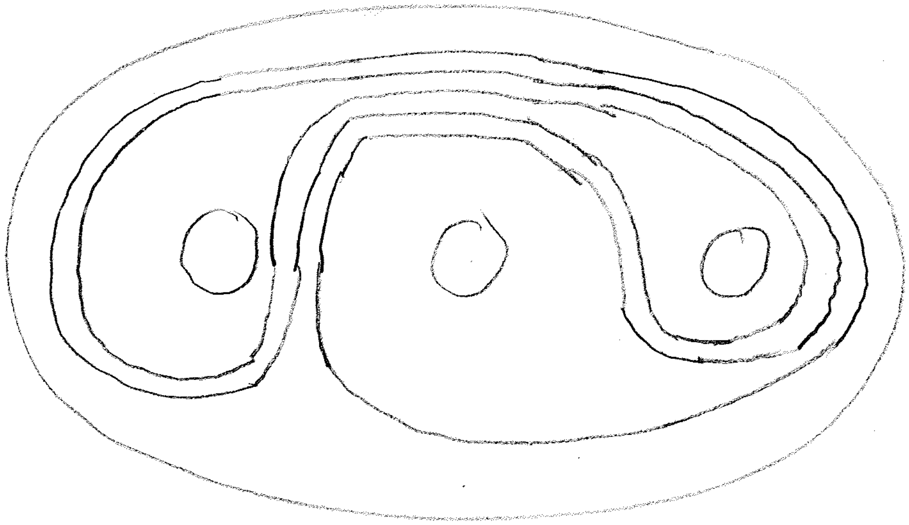
Train tracks: a trivalent graph $\tau \subseteq S$ with a well defined tangent line at each vertex:

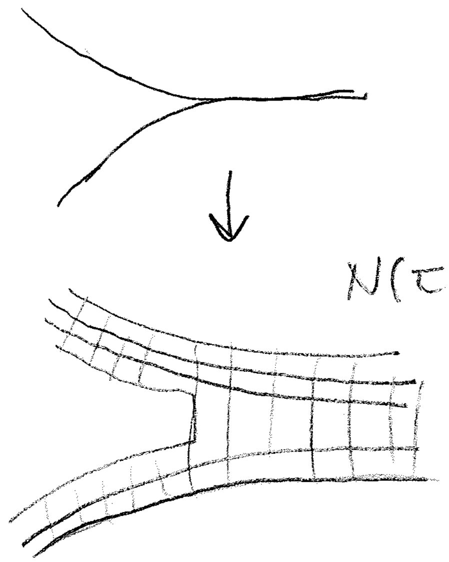


Can be used to encode simple closed curves via weights sat



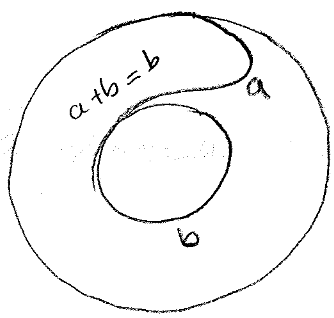
$a + b = c$



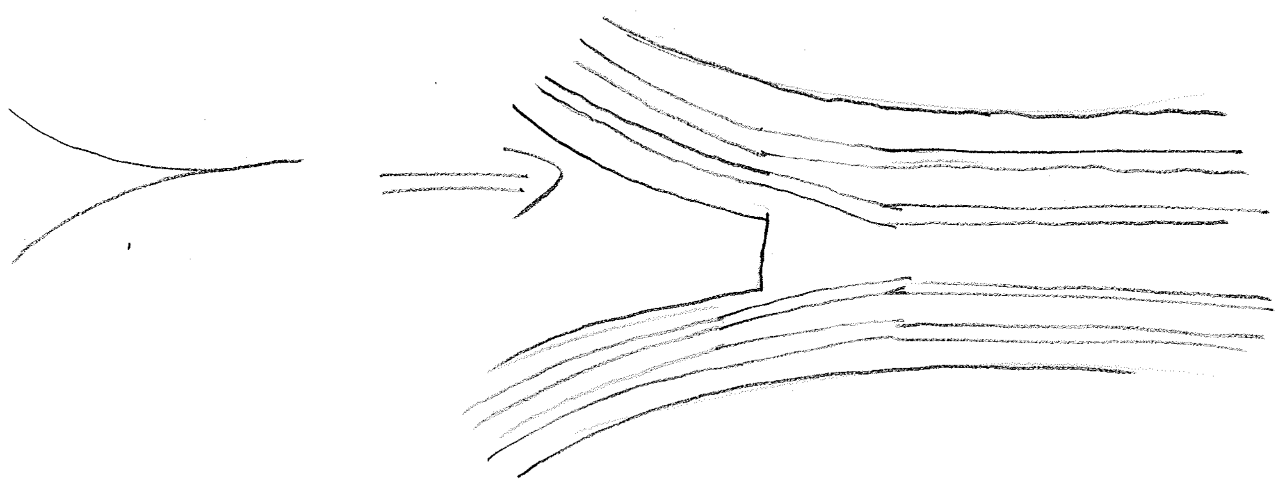


[Comment on many uses.]

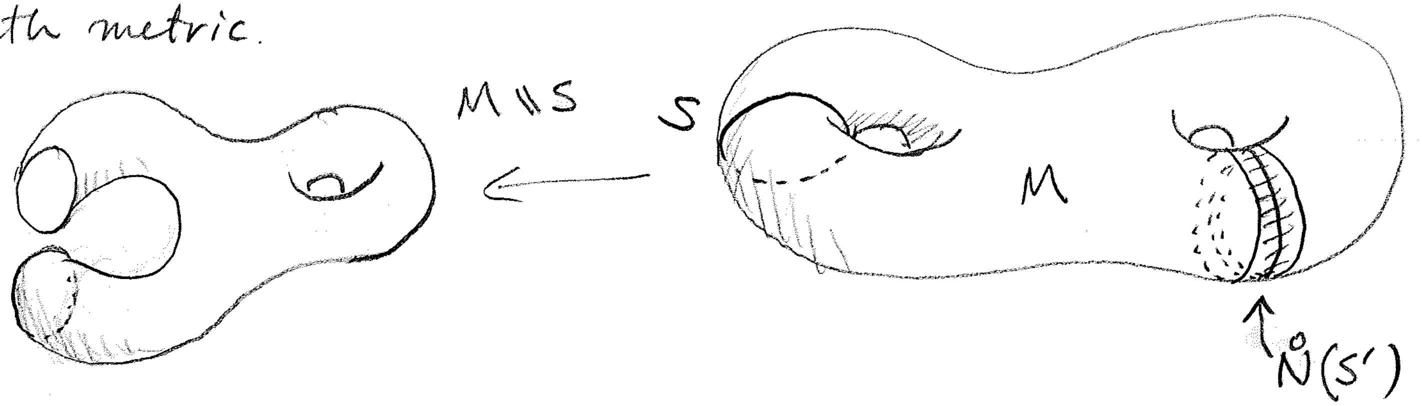
Some τ don't have any weights with full support:



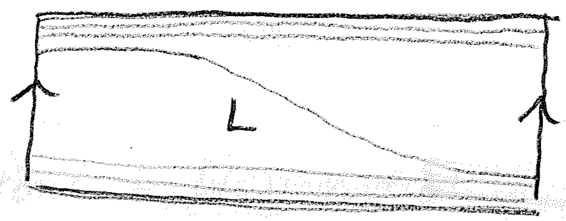
Still can <u>always</u> use to build a lamination: Replace each track segment s by $s \times K$ where $K \subseteq [0,1]$ is a Cantor set.



If $S \subseteq M$ has codim 1, then $M \setminus\!\setminus S$ is the completion of $M \setminus S$ with respect to its internal path metric.

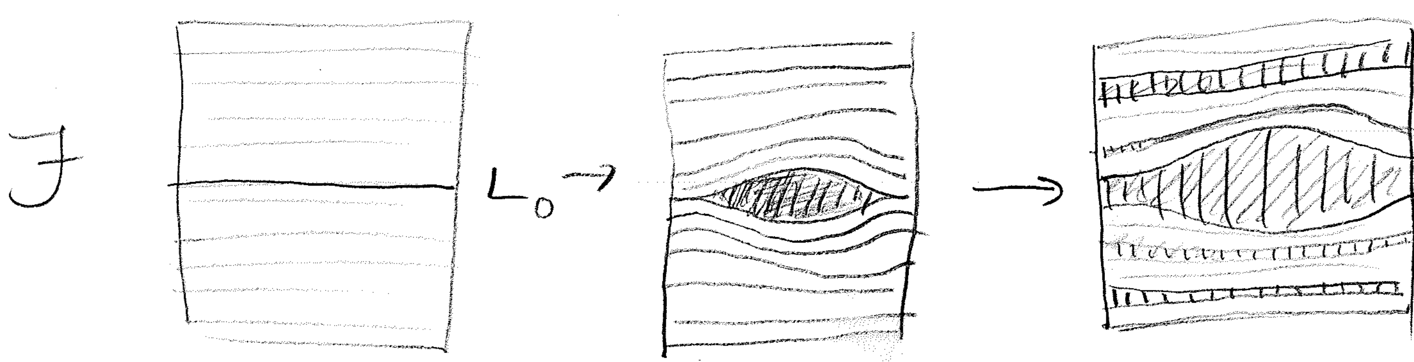


Ex: $\Lambda = \partial A \cup L$ $\qquad A \setminus\!\setminus \Lambda \cong \mathbb{R} \times I$.



If $\Lambda \subseteq M$ is a lamination and every comp of $M \setminus\!\setminus \Lambda$ is a product then can extend Λ to a foliation $\mathcal{F}$.

"Inverse" process: Denjoy blowup.



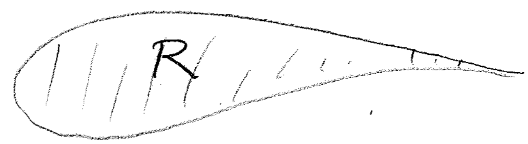
The components of $M \backslash\backslash \Lambda$ are the complementary regions.

Def: A lamination Λ of M^3 is essential when

a) No leaf is S^2 and no torus leaf bounds a solid torus.

b) Each comp. region R is irreducible, ∂R is incomp. in R, and ∂R is end incompressible:

No ess. ideal monogens



Ex: 1) Λ = incomp surface.

2) Λ = taut fol where M is not covered by $S^2 \times S^1$.

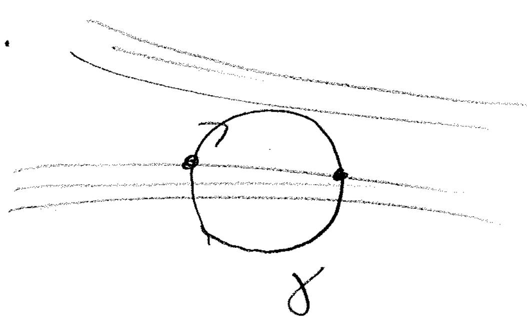
[Gabai–Oertel] Λ an ess. lam of M^3 not cov. by $S^2 \times S^1$.

Then 1) M is irred.

2) all leaves of Λ are incomp.

3) Any tight clsd transv γ to Λ is $\neq 1$ in $\pi_1 M$.

tight: No comp of $\gamma \cap R$ is homotopic, rel end pts, into ∂R.



An ess. lamination has a leaf space $\widetilde{\mathcal{L}}$ which can be like the leaf space of a taut fol or like a simplical tree.

In "favorable circumstances", an ess. lamination also has a univ. circle.

$\times \mathbb{R}$