

Math 416: HW 8 due Friday, April 6, 2018.

Webpage: <http://dunfield.info/416>

Office hours: Tue 10–11, Wed 3:30–4:30, Thur 2–3 and by appointment. My office is 378 Altgeld.

Problems:

- Let \mathbb{C} denote the field of complex numbers, as discussed in detail in Appendix D of [FIS]. As with any field, we can consider vector spaces, linear transformations, and matrices over \mathbb{C} rather than over our usual field \mathbb{R} .
 - The complex numbers \mathbb{C} can be viewed as a vector space over either \mathbb{C} or \mathbb{R} with the usual scalar multiplication. Prove that \mathbb{C} has dimension 1 as a vector space over \mathbb{C} but has dimension 2 as a vector space over \mathbb{R} . In each case, give an explicit basis.
 - Since \mathbb{R} is a subset of \mathbb{C} , if V is a vector space over \mathbb{C} then it is also a vector space over \mathbb{R} : just use the same scalar multiplication but restricted to scalars in \mathbb{R} . If V has dimension n as a vector space over \mathbb{C} , prove that it has dimension $2n$ as a vector space over \mathbb{R} . Hint: Use Theorem 2.19 from [FIS] to reduce to the case where V is just \mathbb{C}^n .
 - Diagonalize the following matrices over \mathbb{C} by giving a $Q \in M_{2 \times 2}(\mathbb{C})$ so that $Q^{-1}AQ$ is diagonal.

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$$

- Section 5.1 of [FIS], Problem 1.
- Section 5.2 of [FIS], Problem 1.
- Section 5.2 of [FIS], Problem 2, parts (e) and (g).
- Section 5.2 of [FIS], Problem 3, parts (a) and (d).
- Prove that similar matrices have the same characteristic polynomial.
- Section 5.2 of [FIS], Problem 7.
- If A is a square matrix prove that A and A^t have the same eigenvalues. Do they have the same eigenvectors? Either prove they do, or give a counterexample.
- Suppose that A in $M_{n \times n}(\mathbb{R})$ has two distinct eigenvalues λ_1 and λ_2 , and that $\dim(E_{\lambda_1}) = n - 1$. Prove that A is diagonalizable.
- Section 5.3 of [FIS], Problem 6.