Math 416: HW 7 due Friday, March 30, 2018.

Webpage: http://dunfield.info/416

Office hours: Tue 10–11, Wed 3:30–4:30, Thur 2–3 and by appointment. My office is 378 Altgeld.

Problems:

1. Prove the following result that was used in class. Suppose E is the elementary matrix obtained from I_n by the row operation R, that is, $I_n \xrightarrow{R} E$. Prove that for all $A \in M_{n\times n}(\mathbb{R})$ one has $A \xrightarrow{R} EA$. Said another way, left-multiplication by E implements the row operation that built E in the first place.

2. Prove that if $A, B \in M_{n\times n}(\mathbb{R})$ are similar matrices then $\det(A) = \det(B)$.

3. A matrix $Q \in M_{n\times n}(\mathbb{R})$ is called orthogonal if $QQ^t = I_n$.

   (a) Prove that if Q is orthogonal then $\det(Q) = \pm 1$.

   (b) Give examples of orthogonal matrices for $n = 2$ with both possible values of the determinant.

4. Suppose $A, B \in M_{n\times n}(\mathbb{R})$ satisfy $AB = I_n$.

   (a) Use the determinant to prove that A is invertible.

   (b) Prove or disprove: $B = A^{-1}$.

5. Section 5.1 of [FIS], Problem 2 parts (a) and (c).

6. Let T be a linear operator on a finite-dimensional vector space V.

   (a) Show that T is invertible if and only if 0 is not an eigenvalue of T.

   (b) If T is invertible, show that λ^{-1} is an eigenvalue of T^{-1} if and only if λ is an eigenvalue of T.

7. Suppose $T\colon V \to V$ is a linear operator with V finite-dimensional. Suppose $v \in V$ is an eigenvector of T with eigenvalue λ. As usual, $T^m\colon V \to V$ denotes composition of T with itself m times. Prove that v is also an eigenvector for T^m and give a formula for the corresponding eigenvalue.

8. Section 5.1 of [FIS], Problem 3(a).

9. Section 5.1 of [FIS], Problem 4 parts (b) and (h).