Math 418: HW 5 due Wednesday, March 4, 2015.

Takehome midterm #1: The median and median on this exam were both 41/50 and the variance was 5.5. Solutions have been posted on the course webpage. There is also now a link at the end of the "Grading" section of the webpage where you can see all your HW and exam scores as well as your overall average in the course.

In class Midterm Exam: The in class midterm will be Monday, March 9. The test will cover Chapters 8, 9, and 13 of the text. It will be closed book, but you can bring one sheet of standard-sized paper on which you may write/copy/print anything you think might be helpful to you. You can use both sides, but must be able to read it without special equipment, so no jeweler's loups.

Webpage: http://dunfield.info/418

Office hours: Mondays and Tuesdays from 2:30–3:30pm and by appointment.

1. Let K_1 and K_2 be subfields of some ambient field L which both contain a subfield F. Suppose that each K_i is the splitting field of a polynomial $f_i \in F[x]$. Prove that $K_1 \cap K_2$ is also the splitting field of some polynomial in $F[x]$.

   Hint: Problem 5 from HW #4.

2. Find all irreducible polynomials of degree 1, 2, and 4 over $\mathbb{F}_2$. Check that their product is $x^{16} + x$.

3. Suppose K is an extension of a perfect field F. Suppose that $f(x) \in F[x]$ has no repeated irreducible factors in $F[x]$. Prove that it also has no repeated irreducible factors in $K[x]$.

4. A field element ζ is called a root of unity if $\zeta^n = 1$ for some $n > 0$. Using the results of Section 13.6, prove that if K is a finite extension of $\mathbb{Q}$ then it contains only finitely many roots of unity.