Math 526: HW 6 due Wednesday, November 19, 2014.

See advertisement for Math 527 on the back.

Note: No class on Friday, November 14, 2014.

1. Hatcher §4.2: #31.

2. Hatcher §4.2: #39.

3. Hatcher §4.3: #2.

4. Prove part (d) of the below. Parts (a-c) were on HW #3.

   Poincaré duality for 3-manifolds. Let M be a closed connected orientable 3-manifold. The only interesting case of Poincaré duality here is that $H^1(M,\mathbb{Z})$ is isomorphic to $H_2(M,\mathbb{Z})$. Fill in the following outline for part of a geometric proof (all (co)homology has coeffs in $\mathbb{Z}$).

   (a) Prove that any class x in $H_1(M)$ can be represented by an oriented embedded circle.

   (b) Prove that any class y in $H_2(M)$ can be represented by an oriented embedded surface. That is, there is an embedded surface $S \subset M$ with $i_*([S]) = y$.

   (c) There is a bilinear pairing $H_1(M) \otimes H_2(M) \to \mathbb{Z}$, namely the intersection product (also called the homology cap product). Show that this map is injective.

   (d) Prove that $H_2(M) \to H_1(M)^* \cong H^1(M)$ is surjective, completing the proof of Poincaré duality. Hint: Use that $H^1(M;\mathbb{Z}) \cong [M,\ S^1]$.

5. Hatcher §4.3: #4.

6. Hatcher §4.3: #6.

Mathematics 527 — Homotopy Theory
(Spring 2015, 3:00 MWF, 341 Altgeld)

(This is a revised syllabus.)

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Prerequisites: Math 526, or instructor consent.

Texts: There are no standard texts on all of this material. References may include

- Quillen, *Homotopical algebra*, Springer LNM 43, (1967).
- Hovey, *Model categories*, AMS Math Surveys 63, (1999).
- Goerss and Jardine, *Simplicial homotopy theory*, Birkhäuser, (1999).
- Joyal, *Theory of quasicategories and its applications*, `http://mat.uab.cat/~kock/crm/hocat/advanced-course/Quadern45-2.pdf`
- Joyal, *Notes on quasicategories*, `www.math.uchicago.edu/~may/IMA/Joyal.pdf`
- Lurie, *Higher topos theory*, Princeton AM 170, (2009), `http://www.math.harvard.edu/~lurie/papers/croppedtopoi.pdf`

Course schedule: This course is an introduction to the basic concepts of modern homotopy theory. Homotopy theory began as the study of continuous deformation of continuous maps between topological spaces; it now encompasses an array of concepts including those of derived functors and higher category theory.
After a brief review of the classical homotopy theory of topological spaces (as covered in 526), we'll discuss

- model structures in the sense of Quillen; the standard model structure on spaces;
- derived functors; homotopy limits and colimits;
- localization of model structures; Postnikov approximation;
- the homotopy theory of G-spaces (G a group); the equivalence of G-spaces and spaces over BG.

The second part of the course will deal a selection of other topics, which might include one or more of the following, depending on interest:

- simplicial homotopy theory;
- stable homotopy theory and spectra;
- introduction to ∞-categories;
- quasi-categories and their homotopy theory.