Math 526: HW 3 due Wednesday, October 8, 2014.

1. Hatcher §3.3: #6.

2. Hatcher §3.3: #10.

3. Hatcher §3.3: #25.

4. Hatcher §3.3: #26.

5. Let M be a compact connected 3-manifold with a simplicial triangulation $\mathcal{T}$. Prove that for every k-simplex $\sigma \in \mathcal{T}$ the dual "cell" is really a cell. That is, prove that $(\overline{D}(\sigma), \dot{D}(\sigma)) \cong (B^{3-k}, \partial B^{3-k})$.

6. **Poincaré duality for 3-manifolds.** Let M be a closed connected orientable 3-manifold. The only interesting case of Poincaré duality here is that $H^1(M, \mathbb{Z})$ is isomorphic to $H_2(M, \mathbb{Z})$. Fill in the following outline for part of a geometric proof (all (co)homology has coeffs in $\mathbb{Z}$).

   (a) Prove that any class x in $H_1(M)$ can be represented by an oriented embedded circle.

   (b) Prove that any class y in $H_2(M)$ can be represented by an oriented embedded surface. That is, there is an embedded surface $S \subset M$ with $i_*([S]) = y$.

   (c) There is a bilinear pairing $H_1(M) \otimes H_2(M) \to \mathbb{Z}$, namely the intersection product (also called the homology cap product). If x is represented by an embedded circle and y is represented by an embedded surface with x and y intersecting transversely, this is just the number of times x crosses y, counted with signs. This gives a map from $H_2(M) \to H_1(M)^* = \operatorname{Hom}(H_1(M), \mathbb{Z})$. Show that this map is injective. (You may assume the intersection product is well-defined.)

   In a later HW you will show that $H_2(M) \to H_1(M)^* \cong H^1(M)$ is surjective, completing the proof of Poincaré duality in this case.