Math 518: HW 12 due Wednesday, December 3, 2014.

1. Problem 17–1 of Lee on page 464.

2. Suppose S is an embedded submanifold of M. A retract is a smooth map $R\colon M \to S$ which is the identity on S. Prove that $R^*\colon H^*(S) \to H^*(M)$ is injective.

3. Let $M = M_1 \times M_2$ and let $P_i\colon M \to M_i$ be the natural projections.

   Prove or disprove: Each $P_i^*\colon H^*(M_i) \to H^*(M)$ is injective.

4. Let $\Omega_c^*(M)$ denote the subalgebra of differential forms with compact support.

   (a) Show you can define $H_c^*(M)$ using $\Omega_c^*(M)$ analogously to how $H^*(M)$ is defined from $\Omega^*(M)$.

   (b) If M is connected and non-compact, prove that $H_c^0(M) = 0$.

   (c) Prove or disprove: A smooth map $F\colon M \to N$ induces a homomorphism $F^*\colon H_c^*(N) \to H_c^*(M)$.

5. In the notes for Wednesday, November 18, complete the proof of the Homotopy Operator Lemma in the case of $\beta = f(x,t)\,dt \wedge dx_1 \wedge \cdots \wedge dx_{k-1}$; see the bottom of page 5 for details.