Math 518: HW 9 due Wednesday, November 5, 2014.

1. Suppose $\alpha_1, \ldots, \alpha_k$ are elements of V^*. Show that they are linearly dependent if and only if $\alpha_1 \wedge \cdots \wedge \alpha_k = 0$ in $\Lambda^k(V)$.

2. Let η be the element of $\Omega^2(S^2)$ defined on page 3 of the lecture notes for Oct 29. Using your choice of chart (U, ϕ) from Problem 4 on HW 8, give the explicit coordinate form for $(\phi^{-1})^*(\eta)$ in $\Omega^2(\phi(U))$. That is, find the function $f\colon \phi(U) \to \mathbb{R}$ so that $(\phi^{-1})^*(\eta) = f(x_1, x_2)\, dx_1 \wedge dx_2$.

3. Suppose $F\colon M \to N$ is smooth. If $\omega \in \Omega^k(N)$ and $\eta \in \Omega^\ell(N)$, prove that

$$F^*(\omega \wedge \eta) = (F^*\omega) \wedge (F^*\eta) \quad \text{in } \Omega^{k+\ell}(M).$$

4. In this problem, you'll give two proofs that S^n is orientable.

   (a) Prove the following general fact and then apply it to S^n:

   Suppose M is a smooth manifold that is the union of two connected orientable open submanifolds U and V with $U \cap V$ connected. Prove that M is orientable.

   (b) Generalize the form η in Problem 2 above from S^2 to S^n and show that it defines an orientation form. Hint: There's no need for messy computations here.

5. Prove that any Lie group G is an orientable manifold.

6. Let $\Theta\colon \mathbb{R} \times M \to M$ be a global flow of an orientable manifold M. For each $t \in \mathbb{R}$, prove that the diffeomorphism $\Theta_t\colon M \to M$ is orientation preserving.