## Math 518: HW 6 due Wednesday, October 8, 2014.

1. Consider $S^3$ as the unit sphere in $\mathbb{C}^2$.

   (a) The *Hopf action* of $S^1$ on $S^3$ is defined by $z \cdot w = zw$ where $z \in S^1$ and $w = (w_1, w_2) \in \mathbb{C}^2$. Show that this action is smooth and that its orbits are disjoint unit circles in $\mathbb{C}^2$ whose union is $S^3$.

   (b) Consider the flow $\Theta\colon \mathbb{R} \times S^3 \to S^3$ defined by $\Theta_t(w) = e^{it}w$. Explicitly calculate the infinitesimal generator $V$ of $\Theta$. Your formula for $V$ should associate to each $w \in S^3$ a vector $V_w \in T_w S^3$ where the latter is thought of as a subspace of $T_w\mathbb{C}^2 = \mathbb{C}^2$.

   (c) Prove or disprove: The vector field $V$ from part (b) is nowhere vanishing.

   The quotient of $S^3$ under these actions, that is, the orbit space, turns out to be $S^2$. The map $S^3 \to S^2$ is called the *Hopf fibration*. A superb video visualizing this map is:
   http://www.nilesjohnson.net/hopf.html.

2. (a) Problem 7-13 of Lee on page 172.

   (b) Prove or disprove: $U(n)$ is compact.

3. Problem 9-2 of Lee of page 245.

4. Problem 9-8 of Lee of page 246.

5. Problem 8-20 of Lee on page 202.

6. Problem 8-25 of Lee on page 203.