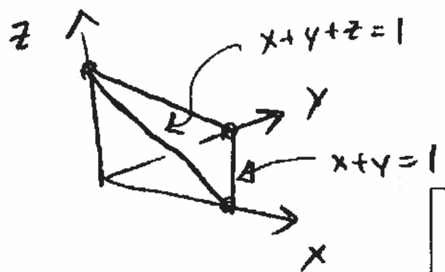
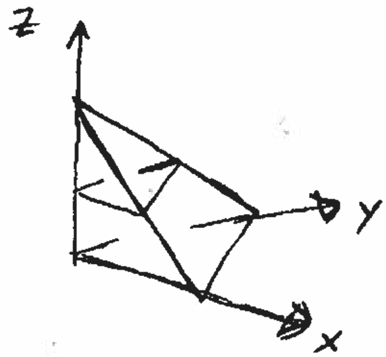
1. Let R be the region in the first octant lying below the plane $x+y+z=1$.

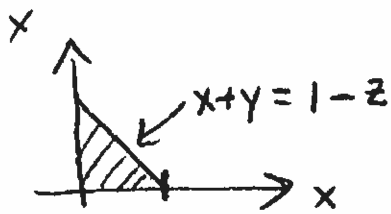
(a) Fill in the limits and integrand of the double integral below so that it computes the volume of R. Be sure to follow the provided order of integration. **(3 points)**



$$\boxed{\text{Volume} = \int_0^1 \int_0^{1-y} 1-x-y \; dx\,dy}$$

(b) Fill in the limits and integrand of the triple integral below so that it computes the volume of R. Be sure to follow the provided order of integration. **(3 points)**





$$\boxed{\text{Volume} = \int_0^1 \int_0^{1-z} \int_0^{1-x-z} 1 \; dy\,dx\,dz}$$

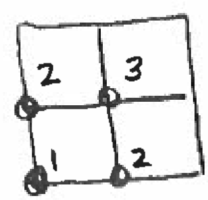
2. Let R be the unit square in the plane with vertices $(0,0)$, $(4,0)$, $(0,4)$, and $(4,4)$. Let f be a continuous function with values as shown in the table at right. Circle the number that is closest to $\iint_R f(x,y)\,dA$:

$f(x,y)$		$x=0$	$x=2$	$x=4$
y	4	1	2	1
	2	2	3	2
	0	1	2	1

0	8	16	32	64	128

(2 points)

① Use 2×2 Riemann Sum:



each square is 2×2 (area = 4)

sample pt in lower left

$= 4\times 1 + 4\times 2 + 4\times 2 + 4\times 3$

$= 4(8) = 32$

② $\iint f\,dA = \text{Area} \times \text{Average} \approx 16 \times \left(\frac{(1+2+1)+(2+3+2)+(1+2+1)}{9}\right)$

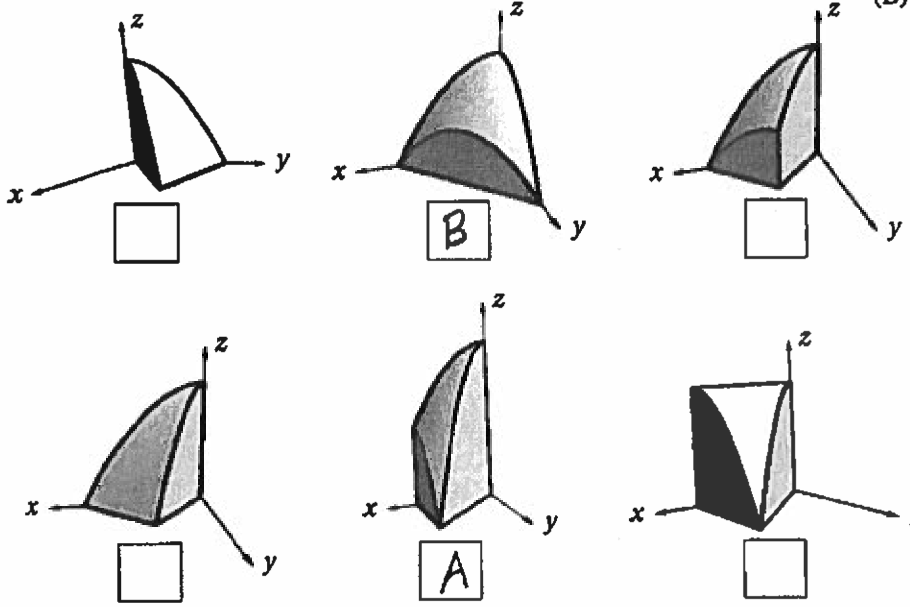
$= 16\left(\frac{15}{9}\right) = 16\cdot \frac{5}{3} = \frac{5}{3} + 15\cdot\frac{5}{3} = 25 + \frac{5}{3} = 26\frac{2}{3}$

which is closest to 32

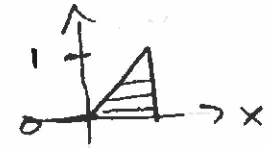
3. Label the boxes below the solid regions corresponding to the two integrals at right. **(2 points each)**

(A) $\displaystyle\int_0^1\int_y^1\int_0^{2-x^2-y^2} f(x,y,z)\,dz\,dx\,dy$

(B) $\displaystyle\int_0^1\int_0^{1-x}\int_0^{1-x^2-y^2} g(x,y,z)\,dz\,dy\,dx$



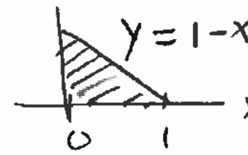
A) Base is



Top is

$2-x^2-y^2$.

B) Base



Top is

$1-x^2-y^2$

4. Let R be the region in the positive octant enclosed by the sphere $x^2+y^2+z^2=4$ and the planes $z=0$, $x=0$, and $y=x$. For **each** integral below, circle "yes" or "no" depending on whether or not it computes $\iiint_R x\,dV$. **(1 point each)**

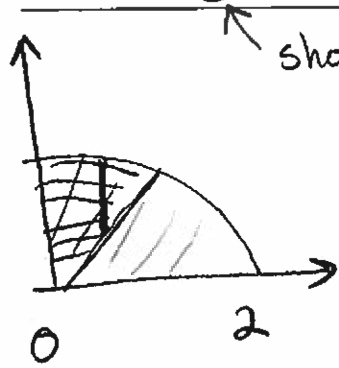
wrong

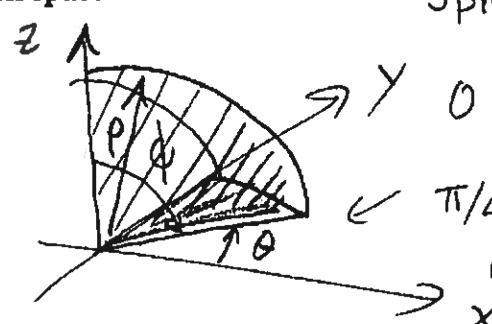
yes no	$\displaystyle\int_0^{\pi/2}\int_0^{\pi/4}\int_0^2 \rho^3\cos\theta\sin^2\phi\,d\rho\,d\theta\,d\phi$	yes no	$\displaystyle\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2\cos\theta\sin\phi\,d\rho\,d\phi\,d\theta$
yes no	$\displaystyle\int_0^{\pi/2}\int_0^2\int_{\pi/4}^{\pi/2} \rho^3\cos\theta\sin^2\phi\,d\theta\,d\rho\,d\phi$	yes no	$\displaystyle\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2\cos\theta\sin\phi\,d\rho\,d\theta\,d\phi$
yes no	$\displaystyle\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} x\,dz\,dy\,dx$	yes no	$\displaystyle\int_0^2\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{4-r^2}} r^2\cos\theta\,dz\,d\theta\,dr$

Wrong

should be X

Scratch Space





Spherical Box

$0 \le \phi \le \pi/2$

$\pi/4 \le \theta \le \pi/2$

$0 \le \rho \le 2$

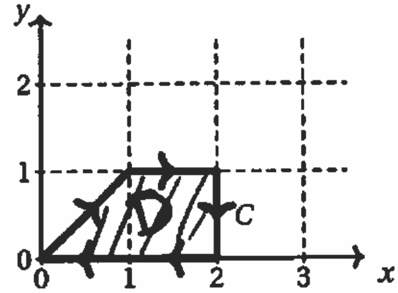
$dV = \rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

$x = \rho\sin\phi\cos\theta$

integrant $\rho^3\sin^2\phi\cos\theta$

5. Let C be the oriented curve shown at right against a dashed grid of unit squares. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F} = \langle y^2 + \cos x,\ x + e^y \rangle$. **(6 points)**

P Q



By Mr. Green:

$\int_C \mathbf{F}\cdot d\mathbf{r} = \boxed{-1/6}$

$$\int_C \vec{F}\cdot d\vec{r} = -\int_{-C} \vec{F}\cdot d\vec{r}$$

$-C$ counter-clockwise

$$= -\iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$$

$$= \iint_D \frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\, dA = \iint_D 2y - 1\, dA$$

$$2y - 1$$

$$= \int_0^1 \int_y^2 2y - 1 \quad dx\, dy$$

$$= \int_0^1 \underbrace{(2y-1)(2-y)}_{-2y^2+5y-2}\, dy = -\frac{2}{3}y^3 + \frac{5}{2}y^2 - 2y \Big|_{y=0}^{1}$$

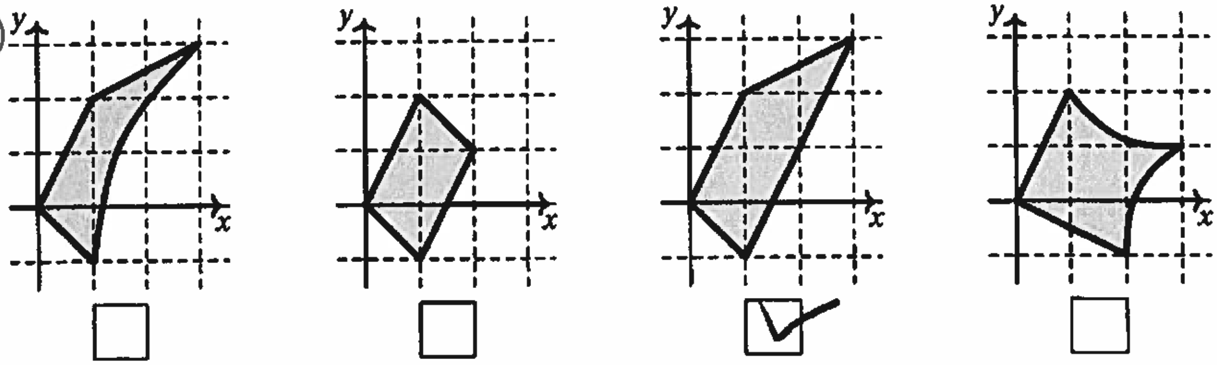
$$= -\frac{2}{3} + \frac{5}{2} - 2 = \frac{-4+15-12}{6} = -\frac{1}{6}$$

6. Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be the transformation $T(u, v) = (\overset{x}{u + v + uv},\ \overset{y}{-u + 2v + 2uv})$. Let $S = \{0 \le u \le 1,\ 0 \le v \le 1\}$ be the unit square in the (u, v)–plane, and let $R = T(S)$ be the region that is the image of S under T.

(a) Check the box below the picture of R drawn against a dashed grid consisting of **unit squares**. **(2 points)**

$T(1,1) = (3,3)$

$T(1,\frac{1}{2}) = (2,1)$



(b) Fill in the limits and integrand of the integral below so that it computes $\iint_R \sqrt{x}\, dA$ as an integral over the square S. **(4 points)**

Jacobian matrix: $\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} 1+v & 1+u \\ -1+2v & 2+2u \end{pmatrix}$

Det is: $(2+2u+2v+2uv) - (-1-u+2v+2uv)$

$= 3+3u$ which is > 0.

$$\iint_R \sqrt{x}\, dA = \int_0^1 \int_0^1 \sqrt{u+v+uv}\ (3+3u)\ du\, dv$$

Scratch Space

7. Let S be the surface in $\mathbb{R}^3$ parameterized by $\mathbf{r}(u, v) = \langle u\cos(v),\ u\sin(v),\ v\rangle$ for $0 \le u \le 1$ and $0 \le v \le \pi$.

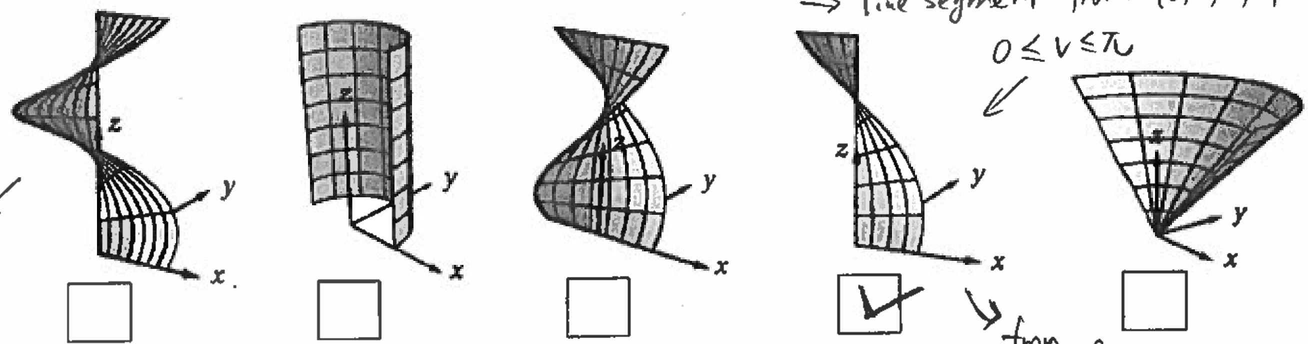
(a) Check the box below the correct picture of S. **(2 points)**

$V = 0 \to (u, 0, 0) \quad 0 \le u \le 1$

$\to$ line segment from $(0,0,0)$ to $(1,0,0)$

$0 \le v \le \pi$

from 0 to 2π



from 0 to π

(b) Evaluate the integral $\iint_S y\, dS$. **(? points)**

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 1 \end{vmatrix} = (\sin v, -\cos v, u)$$

$$\Rightarrow |\vec{r}_u \times \vec{r}_v| = \sqrt{\sin^2 v + \cos^2 v + u^2} = \sqrt{1+u^2}$$

$$\iint_S y\, dS = \int_0^1 \int_0^\pi (u \cancel{\cos}\sin v)\sqrt{1+u^2}\, dv\, du$$

$$= \underbrace{\left(\int_0^\pi \sin v\, dv\right)}_{} \cdot \left(\int_0^1 u\sqrt{1+u^2}\, du\right)$$

$$= -\cos v \Big|_0^\pi$$

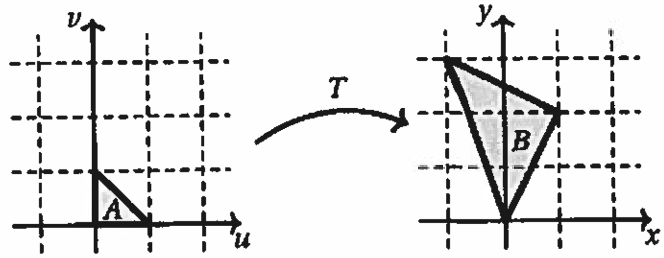
$$= 2$$

$$= \int_0^1 \underbrace{2u}_{\frac{dw}{du}} \underbrace{\sqrt{1+u^2}}_{w}\, du = \int_1^2 w^{1/2}\, dw$$

$$= \frac{2}{3} w^{3/2} \Big|_{w=1}^{w=2}$$

$$\boxed{\iint_S y\, dS = \tfrac{2}{3}\left(2^{3/2} - 1\right)}$$

8. Find a transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ taking the triangle A with vertices $(0,0), (0,1), (1,0)$ to the triangle B with vertices $(0,0), (1,2), (-1,3)$.



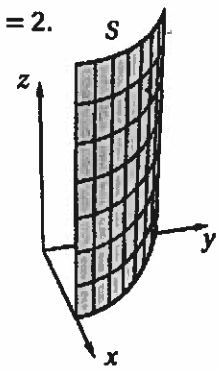
Use linear trans sending $(1,0) \to (1,2)$

$(0,1) \to (-1,3)$

so matrix is $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

$T(u,v) = ($ $u - v$, $2u+3v$ $)$

9. Let S be the portion of the surface $x + y^2 = 1$ in the first octant that lies below the plane $z = 2$.



(a) Parameterize S by $\mathbf{r}\colon D \to \mathbb{R}^3$, being sure to specify the domain D of the parameterization in the (u,v)-plane. **(? points)**

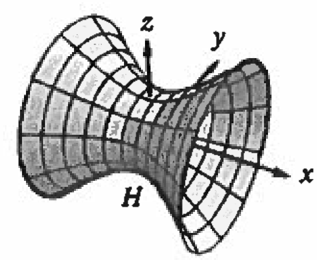
Use $u = y$ and $v = z$ as params,

so $x = 1 - u^2$

$D = \{0 \le u \le 1, 0 \le v \le 2\}$ $\mathbf{r}(u,v) = \langle$ $1 - u^2$, u, v $\rangle$

(b) The integral $\iint_S z\, dS$ is: negative zero positive **(1 point)**

10. Consider the hyperboloid $H = \{y^2 + z^2 = 1 + x^2 \text{ and } -1 \le x \le 1\}$ which is shown below.



(a) Parameterize H by $\mathbf{r}\colon D \to \mathbb{R}^3$, being sure to specify the domain D of the parameterization in the (u,v)-plane. **(? points)**

Params: $u = x$, $v =$ angle about x-axis

Radius when u is fixed is $\sqrt{y^2+z^2} = \sqrt{1+x^2}$

$D = \{-1 \le u \le 1, 0 \le v \le 2\pi\}$

$\mathbf{r}(u,v) = \langle$ u , $\sqrt{1+u^2}\cos v$, $\sqrt{1+u^2}\sin v$ $\rangle$

(b) The integral $\iint_H z\, dS$ is: negative zero positive **(1 point)**

H symmetric with respect to xy-plane.