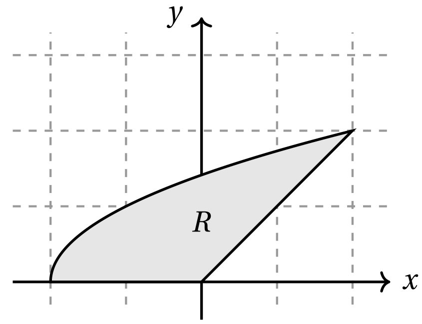
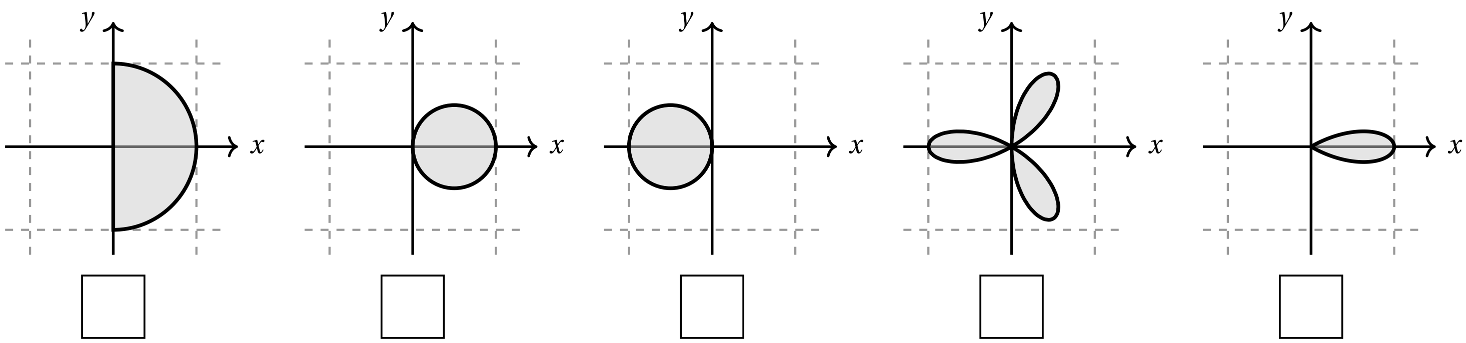
1. Let R be the region shown which is bounded by the curve $y^2 - x - 2 = 0$, the line $y = x$, and x-axis. Evaluate $\iint_R 3y\,dA$. **(4 points)**



$\iint_R 3y\,dA =$

2. The integral $\iint_R 2x^2 + 2y^2 + y\,dA$ has the form $\int_{-\pi/2}^{\pi/2} \int_0^{\cos\theta} ??\,dr\,d\theta$ when converted into polar coordinates.

(a) Mark the box below the picture of the region that represents R. **(2 points)**



(b) Fill in the missing integrand to convert this integral into polar coordinates. **(2 points)**

$$\iint_R 2x^2 + 2y^2 + y\,dA = \int_{-\pi/2}^{\pi/2} \int_0^{\cos\theta} \boxed{}\, dr\,d\theta.$$

Scratch Space

3. Consider the region R in the positive octant bounded by the cone $z = \sqrt{x^2 + y^2}$ and the planes $z = 1$, $x = 0$, and $y = x$. In **each column** below, exactly one of the iterated integrals computes $\iiint_R x\,dV$. Determine which are the correct answers and mark the boxes next to them. **(2 points each)**

☐ $\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{\sec\phi} \rho^3 \sin^2\phi\cos\theta\ d\rho\, d\phi\, d\theta$	☐ $\int_0^1\int_{\pi/4}^{\pi/2}\int_0^{z^2} r^2\cos\theta\ dr\, d\theta\, dz$
☐ $\int_{\pi/4}^{\pi/2}\int_0^{\pi/4}\int_0^{\sec\phi} \rho^3 \sin^2\phi\cos\theta\ d\rho\, d\phi\, d\theta$	☐ $\int_0^1\int_{\pi/4}^{\pi/2}\int_0^{z} r\cos\theta\ dr\, d\theta\, dz$
☐ $\int_{\pi/4}^{\pi/2}\int_0^{\pi/4}\int_0^{\sec\phi} \rho^2 \sin^2\phi\cos\theta\ d\rho\, d\phi\, d\theta$	☐ $\int_0^1\int_{\pi/4}^{\pi/2}\int_0^{z} r^2\cos\theta\ dr\, d\theta\, dz$

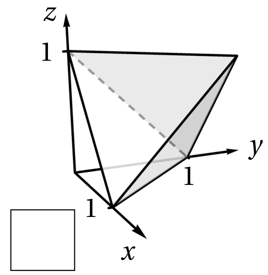
Scratch Space

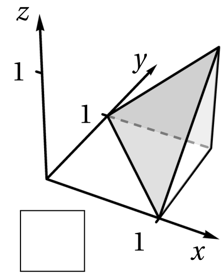
4. A rectangular metallic plate R is placed in the plane with vertices at $(-2,-1)$, $(-2,1)$, $(2,-1)$, and $(2,1)$. The density (in g/cm^2) of the plate, $\rho(x,y)$, at various points is shown in the table, where x and y are measured in cm. Circle the best estimate for the mass of the plate. **(2 points)**

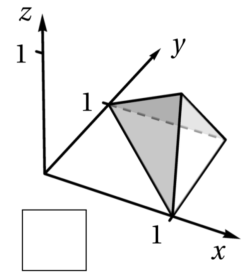
$\rho(x,y)$		x	
		-1	1
	$1/2$	4	7
y	$-1/2$	1	3

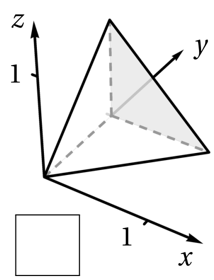
Mass of $R \approx$ | 0 | 4 | 15 | 30 | 46 | 60 | 78 | grams.

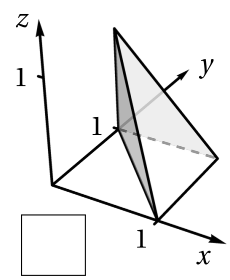
5. The integral of the function $f(x,y,z) = 2x$ over a region R is computed by $\int_0^1 \int_0^y \int_0^{y-x} 2x \, dz \, dx \, dy$. Mark the box below the picture of the region R. **(2 points)**

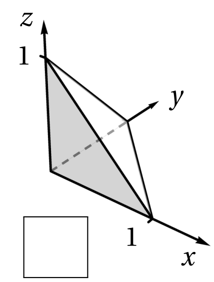










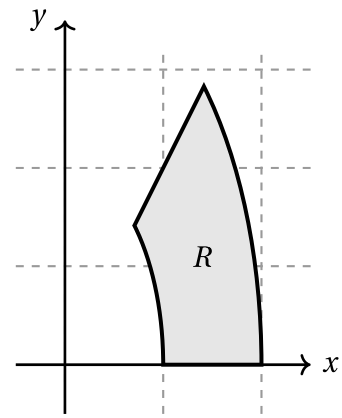


Scratch Space

6. Suppose R is the region in the first quadrant between the ellipses $x^2 + \frac{y^2}{4} = 1$ and $x^2 + \frac{y^2}{4} = 4$ and the lines $y = 0$ and $y = 2x$ shown at the right. Using the transformation

$$T(u, v) = \left\langle u\cos(v),\ 2u\sin(v)\right\rangle$$

find the integrand and limits of integration expressing the integral $\iint_R x\, dA$ as an iterated integral over a subset S in the uv–plane with $T(S) = R$. **(5 points)**

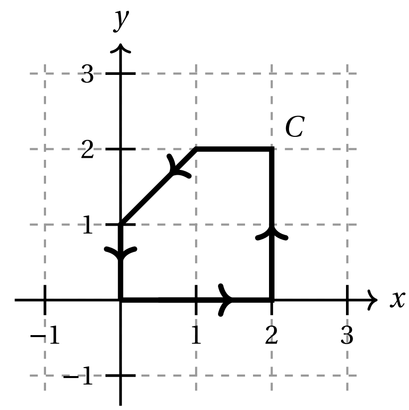


$$\iint_R x\, dA = \boxed{\int \int \qquad\qquad\qquad\qquad du\,dv}$$

Note: The order of integration is already determined.

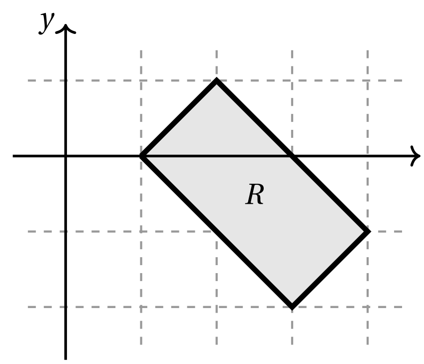
Scratch Space

7. Compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = \langle y + 2\cos(x),\ 3x + e^{y^2}\rangle$ and C is the oriented curve shown.
(5 points)



$\int_C \mathbf{F}\cdot d\mathbf{r} =$ ☐

8. Let R be the rectangle whose vertices are $(1,0)$, $(2,1)$, $(3,-2)$, and $(4,-1)$ shown at the right.



(a) Exactly one of the following defines a transformation $T(u,v)$ from the uv–plane to the xy–plane with $T(S) = R$, where $S = \{(u,v) \mid 0 \le u \le 1, 0 \le v \le 1\}$. Circle the correct formula for $T(u,v)$. **(2 points)**

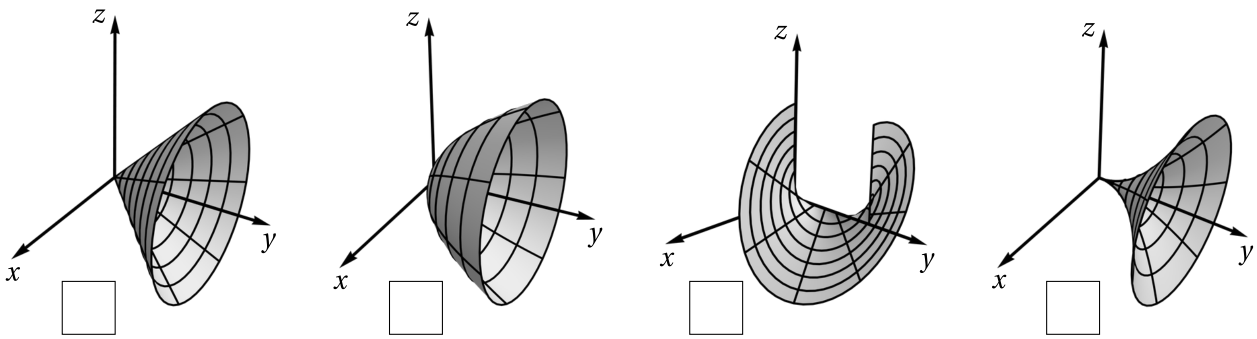
$\langle 2u+3v,\ u-2v\rangle$	$\langle 2u+4v,\ u-v\rangle$	$\langle 2u+4v+1,\ u-v\rangle$
$\langle 2u+3v+1,\ u-2v\rangle$	$\langle u+2v,\ u-2v\rangle$	$\langle u+2v+1,\ u-2v\rangle$

(b) $\iint_R y\,dA$ is | negative zero positive | **(1 point)**

Scratch Space

9. Consider the surface S parameterized by $\mathbf{r}(u,v) = \langle u^2 \sin v,\ u,\ u^2 \cos v \rangle$ for $0 \le u \le 1$ and $0 \le v \le 2\pi$.

(a) Mark the box below the best picture of S. **(1 point)**



(b) Circle the correct formula for $\mathbf{r}_u \times \mathbf{r}_v$. **(2 points)**

$\langle u^2 \sin v,\ u^3,\ u^2 \cos v \rangle$	$\langle u \cos v,\ u^2,\ u \sin v \rangle$	$\langle -u^2 \sin v,\ 2u^3,\ -u^2 \cos v \rangle$	$\langle -u \cos v,\ 2u^2,\ -u \sin v \rangle$

(c) Circle the integrand for the integral $\int_0^1 \int_0^{2\pi} g(u,v)\, dv\, du$ that computes the surface area of S. **(2 points)**

$g(u,v) =$

$\sqrt{u^4 + u^6}$	$\sqrt{u^2 + u^4}$	$\sqrt{4u^4 + 4u^6}$	$\sqrt{4u^2 + 4u^4}$	$\sqrt{u^4 + 4u^6}$	$\sqrt{u^2 + 4u^4}$

(d) $\iint_S xz\, dS$ is

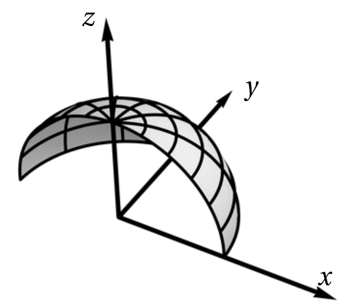
negative	zero	positive

(1 point)

Scratch Space

10. Parameterize each of the surfaces below with a function $\mathbf{r}(u, v)$. Be sure to specify the domain D of your parameterization.

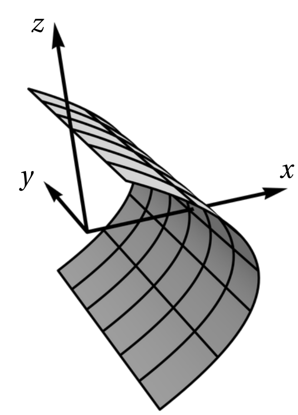
(a) The portion of the sphere $x^2 + y^2 + z^2 = 4$ where $y \geq 0$ and $z \geq 0$. **(3 points)**



$\mathbf{r}(u, v) = \Big\langle \quad , \quad , \quad \Big\rangle$

$D = \Big\{ (u, v) \;\Big|\; \quad \leq u \leq \quad , \quad \leq v \leq \quad \Big\}$

(b) The part of the graph $x = 1 - z^2$ where $x \geq 0$ and $-2 \leq y \leq 2$. **(4 points)**



$\mathbf{r}(u, v) = \Big\langle \quad , \quad , \quad \Big\rangle$

$D = \Big\{ (u, v) \;\Big|\; \quad \leq u \leq \quad , \quad \leq v \leq \quad \Big\}$