1. Let R be the region in the first octant lying below the plane $x + y + z = 1$.

(a) Fill in the limits and integrand of the double integral below so that it computes the volume of R. Be sure to follow the provided order of integration. **(3 points)**

$$\text{Volume} = \int \quad \int \qquad\qquad\qquad\qquad dx\,dy$$

(b) Fill in the limits and integrand of the triple integral below so that it computes the volume of R. Be sure to follow the provided order of integration. **(3 points)**

$$\text{Volume} = \int \quad \int \quad \int \qquad\qquad\qquad\qquad dy\,dx\,dz$$

2. Let R be the unit square in the plane with vertices $(0,0)$, $(4,0)$, $(0,4)$, and $(4,4)$. Let f be a continuous function with values as shown in the table at right. Circle the number that is closest to $\iint_R f(x,y)\,dA$:

0	8	16	32	64	128

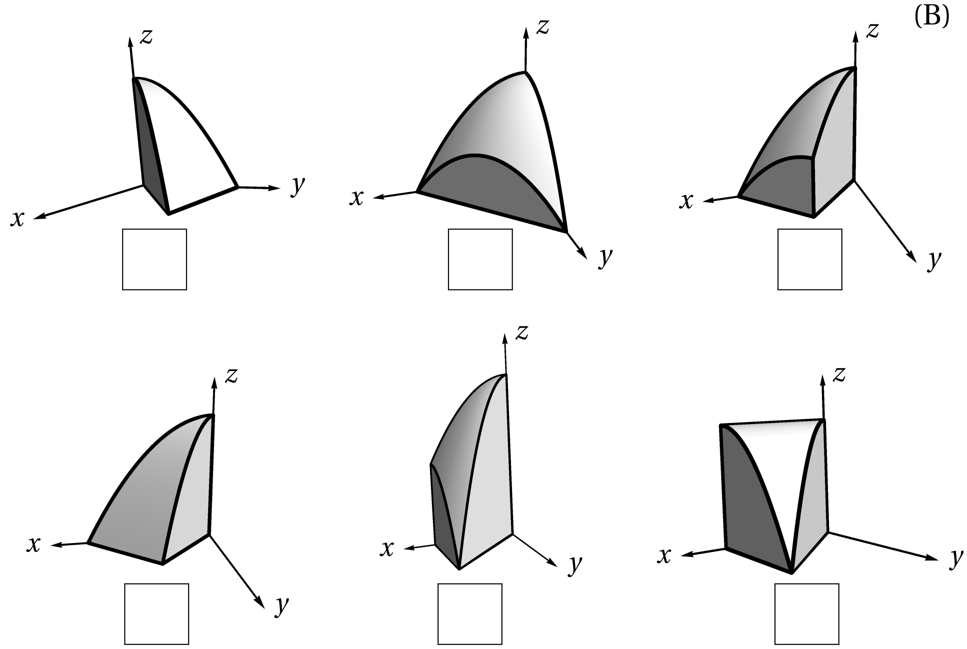
(2 points)

$f(x,y)$		$x=0$	$x=2$	$x=4$
y	4	1	2	1
	2	2	3	2
	0	1	2	1

3. Label the boxes below the solid regions corresponding to the two integrals at right. **(2 points each)**

(A) $$\int_0^1 \int_y^1 \int_0^{2-x^2-y^2} f(x,y,z)\, dz\, dx\, dy$$

(B) $$\int_0^1 \int_0^{1-x} \int_0^{1-x^2-y^2} g(x,y,z)\, dz\, dy\, dx$$



4. Let R be the region in the positive octant enclosed by the sphere $x^2+y^2+z^2=4$ and the planes $z=0$, $x=0$, and $y=x$. For **each** integral below, circle "yes" or "no" depending on whether or not it computes $\iiint_R x\, dV$. **(1 point each)**

yes no $$\int_0^{\pi/2} \int_0^{\pi/4} \int_0^2 \rho^3 \cos\theta \sin^2\phi \; d\rho\, d\theta\, d\phi$$

yes no $$\int_0^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^2 \rho^2 \cos\theta \sin\phi\, d\rho\, d\phi\, d\theta$$

yes no $$\int_0^{\pi/2} \int_0^2 \int_{\pi/4}^{\pi/2} \rho^3 \cos\theta \sin^2\phi \; d\theta\, d\rho\, d\phi$$

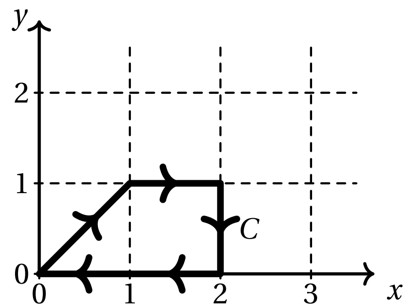
yes no $$\int_0^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^2 \rho^2 \cos\theta \sin\phi \; d\rho\, d\theta\, d\phi$$

yes no $$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} x\, dz\, dy\, dx$$

yes no $$\int_0^2 \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{4-r^2}} r^2 \cos\theta\, dz\, d\theta\, dr$$

Scratch Space

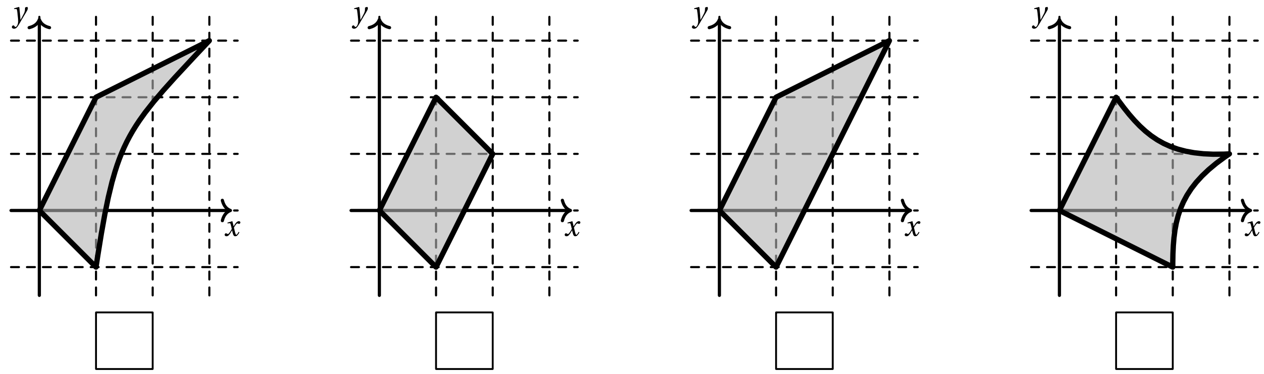
5. Let C be the oriented curve shown at right against a dashed grid of unit squares. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F} = \left\langle\, y^2 + \cos x,\ x + e^y \,\right\rangle$. **(6 points)**



$\int_C \mathbf{F} \cdot d\mathbf{r} =$

6. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation $T(u,v) = (u+v+uv,\ -u+2v+2uv)$. Let $S = \{0 \le u \le 1,\ 0 \le v \le 1\}$ be the unit square in the (u,v)–plane, and let $R = T(S)$ be the region that is the image of S under T.

(a) Check the box below the picture of R drawn against a dashed grid consisting of **unit squares**. **(2 points)**



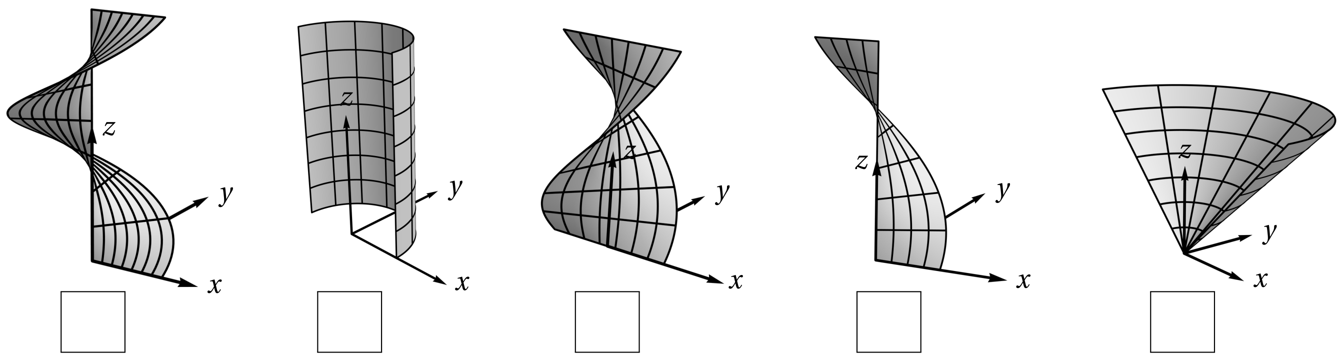
(b) Fill in the limits and integrand of the integral below so that it computes $\iint_R \sqrt{x}\, dA$ as an integral over the square S. **(4 points)**

$$\iint_R \sqrt{x}\, dA = \int \qquad \int \qquad\qquad\qquad\qquad du\, dv$$

Scratch Space

7. Let S be the surface in $\mathbb{R}^3$ parameterized by $\mathbf{r}(u, v) = \langle u\cos(v),\ u\sin(v),\ v\rangle$ for $0 \le u \le 1$ and $0 \le v \le \pi$.

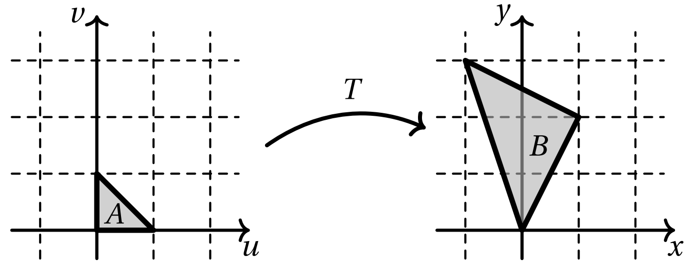
(a) Check the box below the correct picture of S. **(2 points)**



(b) Evaluate the integral $\iint_S y\, dS$. **(6 points)**

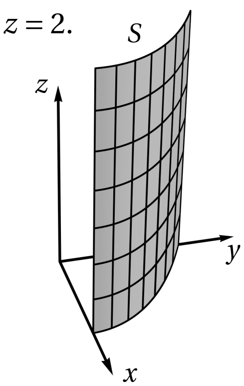
$\iint_S y\, dS =$

8. Find a transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ taking the triangle A with vertices $(0,0), (0,1), (1,0)$ to the triangle B with vertices $(0,0), (1,2), (-1,3)$.
(2 points)



$T(u, v) = \big(\qquad\qquad , \qquad\qquad \big)$

9. Let S be the portion of the surface $x + y^2 = 1$ in the first octant that lies below the plane $z = 2$.



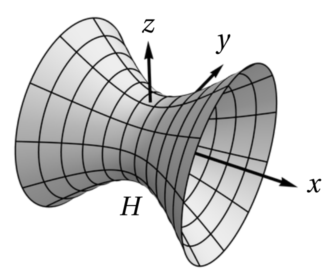
(a) Parameterize S by $\mathbf{r}\colon D \to \mathbb{R}^3$, being sure to specify the domain D of the parameterization in the (u, v)-plane. **(3 points)**

$D = \Big\{ \qquad\qquad \Big\}$ $\quad \mathbf{r}(u, v) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$

(b) The integral $\iint_S z\, dS$ is: negative zero positive **(1 point)**

10. Consider the hyperboloid $H = \{ y^2 + z^2 = 1 + x^2 \text{ and } -1 \leq x \leq 1 \}$ which is shown below.

(a) Parameterize H by $\mathbf{r}\colon D \to \mathbb{R}^3$, being sure to specify the domain D of the parameterization in the (u, v)-plane. **(3 points)**



$D = \Big\{ \qquad\qquad \Big\}$

$\mathbf{r}(u, v) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$

(b) The integral $\iint_H z\, dS$ is: negative zero positive **(1 point)**