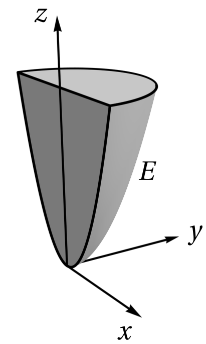
1. Suppose the region E where $x^2 + y^2 \le z \le 4$ and $y \ge 0$ is made of material whose density is given by $\rho(x, y, z) = z$.



(a) Fill in the limits and integrand of the integral below so that it computes the mass of E. **(5 points)**

$$\int_0^4 \int \qquad \int \qquad\qquad\qquad dy\,dx\,dz$$

(b) Circle the center of mass of E, whose coordinates have been rounded to one decimal place. Note: This can be done without evaluating any integrals. **(2 points)**

(0, 0.7, 1)	(0, 0.7, 2)	(0, 0.7, 3)	(0.7, 0, 1)	(0.7, 0, 2)	(0.7, 0, 3)

Scratch Space

2. Consider the transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(u,v) = \left(u^2 - v^2,\ uv\right)$.

(a) Circle the Jacobian of T: **(2 points)**

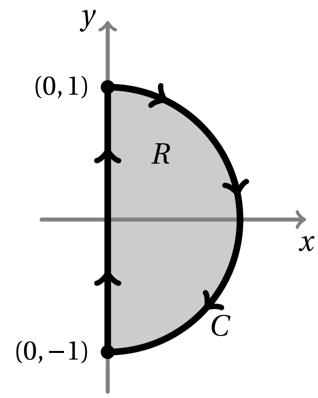
$$\frac{\partial(x,y)}{\partial(u,v)} = \boxed{2u^2+2v^2 \qquad 2u^2-2v^2 \qquad 4uv \qquad 2u+2v}$$

(b) Let S be the square in the (u,v)–plane where $0 \le u \le 1$ and $0 \le v \le 1$. Find the area of its image $T(S)$ in the (x,y)–plane. **(3 points)**

$\boxed{\text{Area}\big(T(S)\big) = \qquad\qquad}$

Scratch Space

3. Consider the vector field $\mathbf{F}(x,y) = \langle ye^x,\ e^x + x\rangle$. Let R be the half disk below, and let C be the boundary of R, oriented as shown.



(a) Use Green's Theorem to compute $\int_C \mathbf{F}\cdot d\mathbf{r}$. **(4 points)**

$\int_C \mathbf{F}\cdot d\mathbf{r} =$

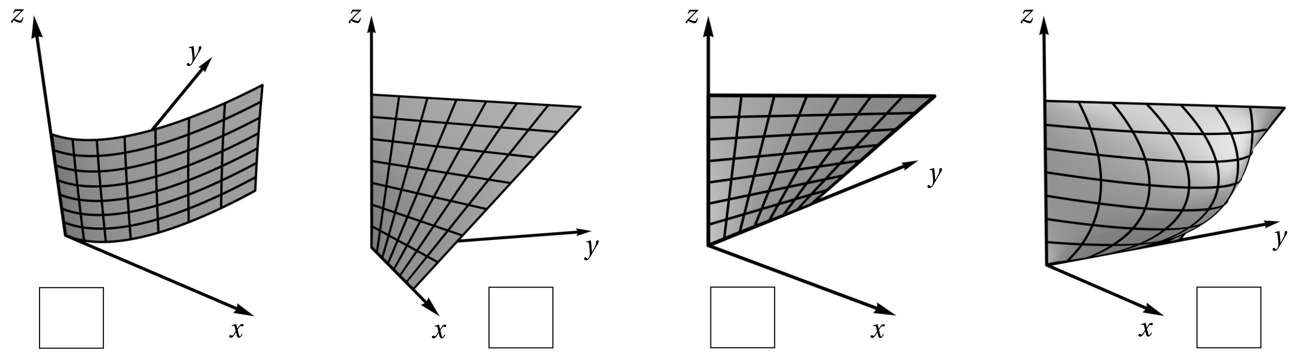
(b) Let C_0 be the round part of C, that is, just the semicircle from $(0,1)$ to $(0,-1)$, not including the y-axis. Compute $\int_{C_0} \mathbf{F}\cdot d\mathbf{r}$. **(2 points)**

$\int_{C_0} \mathbf{F}\cdot d\mathbf{r} =$

Scratch Space

4. Let S be the surface parameterized by $\mathbf{r}(u, v) = \langle uv,\ u,\ v \rangle$ for $0 \leq u \leq 1$ and $0 \leq v \leq 1$.

(a) Mark the box next to the picture of S below. **(2 points)**



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(b) Find a normal vector $\mathbf{v}$ for the tangent plane to S at the point $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{2}\right)$. **(3 points)**

$\mathbf{v} = \langle \quad , \quad , \quad \rangle$

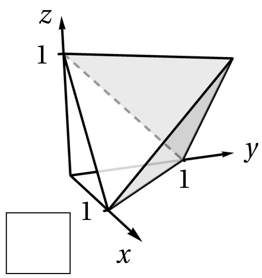
(c) Fill in the integrand below to give a double integral in u and v that evaluates the surface integral $\iint_S x + z \, dS$. Do not evaluate the resulting integral. **(3 points)**

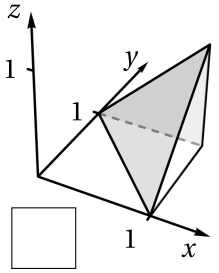
$$\int_0^1 \int_0^1 \qquad\qquad du\, dv$$

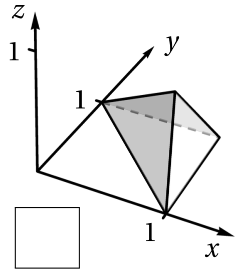
5. Label the boxes next to the solid regions corresponding to the following two integrals: **(2 points each)**

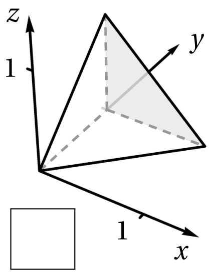
(A) $\displaystyle\int_0^1 \int_0^{1-z} \int_0^{1-y-z} f(x,y,z)\,dx\,dy\,dz$

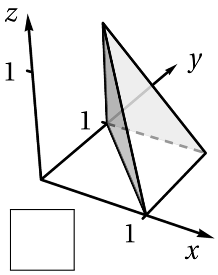
(B) $\displaystyle\int_0^1 \int_z^1 \int_{1-y}^{1-z} g(x,y,z)\,dx\,dy\,dz$

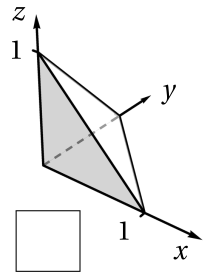












6. Consider the solid $E = \{x^2 + y^2 + z^2 \le 4 \quad \text{and} \quad x \le 0 \quad \text{and} \quad z \le 0\}$.

(a) Check the box next to the correct description of E in terms of spherical coordinates: **(2 points)**

- ☐ $\{0 \le \rho \le 2,\ 0 \le \theta \le \pi,\ \pi/2 \le \phi \le \pi\}$
- ☐ $\{0 \le \rho \le 2,\ 0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi\}$
- ☐ $\{0 \le \rho \le 2,\ 0 \le \theta \le \pi,\ 0 \le \phi \le \pi/2\}$
- ☐ $\{0 \le \rho \le 2,\ \pi/2 \le \theta \le 3\pi/2,\ \pi/2 \le \phi \le \pi\}$
- ☐ $\{0 \le \rho \le 2,\ \pi/2 \le \theta \le 3\pi/2,\ 0 \le \phi \le \pi/2\}$

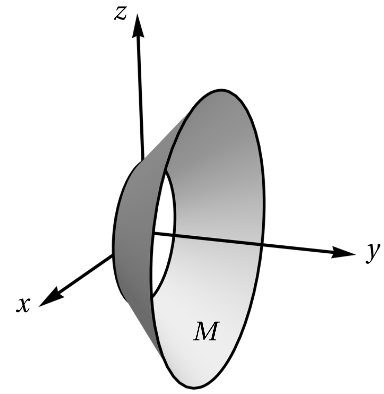
(b) Select the correct integrand that fills in the blank of $\iiint_E z\,dV = \iiint_E \_\_\_\_\_\_\_\_\_\_ \,d\rho\,d\theta\,d\phi$. **(2 points)**

- ☐ $\rho^2 \sin\phi \cos\phi$
- ☐ $\rho^3 \sin\phi \sin\theta$
- ☐ $\rho^2 \sin\phi$
- ☐ $\rho \cos^2\theta$
- ☐ $\rho^3 \sin\phi \cos\phi$
- ☐ $\rho \sin\phi \cos\theta$

7. (a) Consider the rectangle S in $\mathbb{R}^3$ with vertices $(0,0,0)$, $(1,1,0)$, $(1,1,2)$, and $(0,0,2)$. Give a parameterization of S of the form $\mathbf{r}(u,v)$ where $0 \le u \le 1$ and $0 \le v \le 1$. **(2 points)**

$\mathbf{r}(u,v) = \langle \quad , \quad , \quad \rangle$

(b) Let M be the portion of the cone $\sqrt{x^2+z^2} = y+1$ for $0 \le y \le 1$ as shown at right. Parameterize it by $\mathbf{r}\colon D \to \mathbb{R}^3$, being sure to specify the domain D of the parameterization in the (u,v)–plane. **(3 points)**



$\mathbf{r}(u,v) = \langle \quad , \quad , \quad \rangle$

$D = \{ \quad \}$

(c) The surface integral $\iint_M z^2\, dS$ is: | negative zero positive | **(1 point)**

Scratch Space