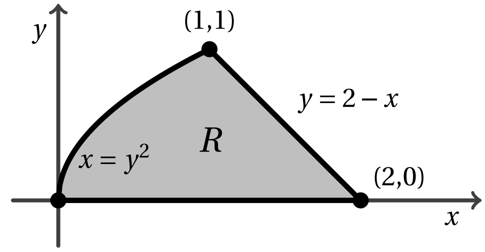
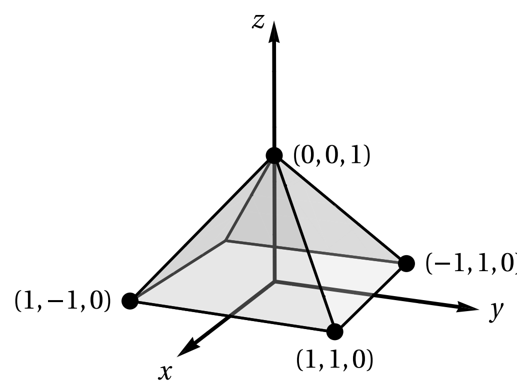
1. Let R denote the shaded region pictured below right. Compute $\iint_R 12y\, dA$. **(4 points)**



$\iint_R 12y\, dA =$

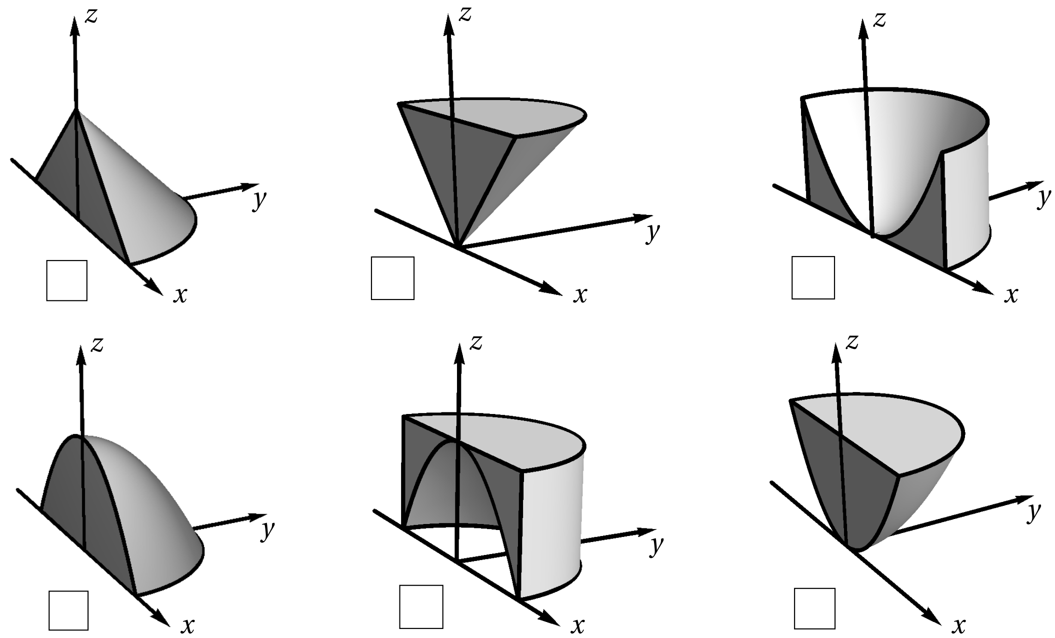
2. Completely setup, but do not evaluate, a triple integral giving the volume of the pyramid shown at right. This pyramid has a square base, and the triangular faces lie in the planes given by $x+z=1, -x+z=1, y+z=1$, and $-y+z=1$. **(5 points)**



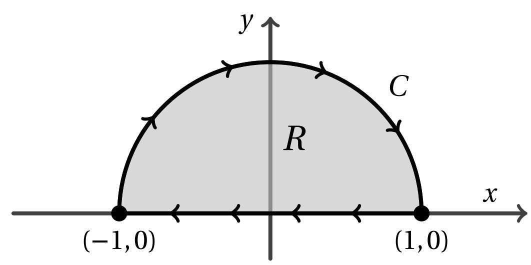
Volume =

3. For each of the integrals: $(a)\ \int_0^{\pi}\int_0^{1}\int_0^{r^2} f(r,\theta,z)\, r\, dz\, dr\, d\theta$ and $(b)\ \int_0^{\pi}\int_0^{1}\int_0^{\sqrt{z}} f(r,\theta,z)\, r\, dr\, dz\, d\theta$

label the solid corresponding to the region of integration below. **(2 points each)**



4. Let $\mathbf{F}(x,y) = \left\langle x-1,\ \cos y + 2x - e^{y^2} \right\rangle$. Let R denote the solid semi-disk shown below right. Let C denote the boundary of the region R.



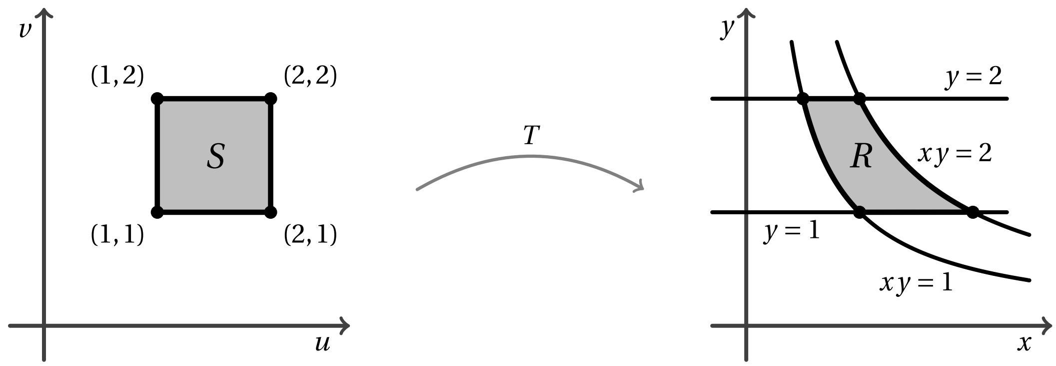
(a) Use Green's Theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C has the orientation shown. **(3 points)**

$\int_C \mathbf{F}\cdot d\mathbf{r} =$

(b) Let D denote the part of the curve C above consisting *only* of the semicircle (*not* the line segment) with the orientation shown. Compute $\int_D \mathbf{F}\cdot d\mathbf{r}$. **(3 points)**

$\int_D \mathbf{F}\cdot d\mathbf{r} =$

5. Let R be the region shown at right.



(a) Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ taking $S = [1,2] \times [1,2]$ to R. **(3 points)**

$T(u, v) = \Big\langle \qquad , \qquad \Big\rangle$

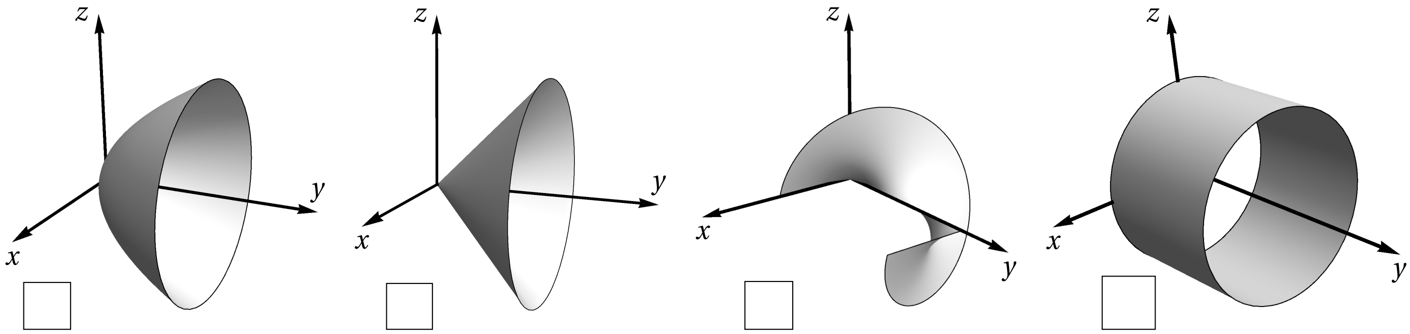
(b) Use your transformation $T(u, v)$ from part (a) to evaluate $\iint_R y^2 \, dA$ via an integral over S. **(4 points)**

Emergency backup transformation: if you can't do (a), pretend you got the answer $T(u, v) = \left(u^5 v, \ \frac{1}{u}\right)$ and do part (b) anyway.

$\iint_R y^2 \, dA =$

6. Let S be the surface parameterized by $\mathbf{r}(u, v) = \langle v\cos u,\ v,\ v\sin u\rangle$ for $0 \le u \le 2\pi$ and $0 \le v \le 1$.

(a) Mark the picture of S below. **(2 points)**



(b) Evaluate the surface integral $\iint_S y\, dS$. **(6 points)**

$\iint_S y\, dS =$

7. Let E be the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 2$. Use **spherical coordinates** to completely setup, but not evaluate, a triple integral which computes the volume of E. **(4 points)**

8. For each surface S in parts (a) and (b), give a parameterization $\mathbf{r}\colon D \to S$. Be sure to explicitly specify the domain D and call your parameters u and v.

(a) The portion of the sphere $x^2 + y^2 + z^2 = 1$ where $x \geq 0$. **(3 points)**

$D = \{ \quad \leq u \leq \quad$ and $\quad \leq v \leq \quad \}$

$\mathbf{r}(u, v) = \langle \quad , \quad , \quad \rangle$

(b) The triangle in $\mathbb{R}^3$ with vertices $(2, 0, 0), (0, 1, 0), (0, 0, 1)$ which lies in the plane $\frac{x}{2} + y + z = 1$. **(2 points)**

$D = \{ \quad \leq u \leq \quad$ and $\quad \leq v \leq \quad \}$

$\mathbf{r}(u, v) = \langle \quad , \quad , \quad \rangle$

(c) Parameterize the cylinder $C = \{x^2 + y^2 = 1\}$ by $\mathbf{r}(u, v) = \langle \cos u,\ \sin u,\ v \rangle$ for $0 \leq u \leq 2\pi$ and v unrestricted. Let M be the part of C above the xy–plane and below the plane $x + z = 2$. Find a region D in $\mathbb{R}^2$ so that $\mathbf{r}(D) = M$. **(1 point)**

$D = \{ 0 \leq u \leq 2\pi$ and $\quad \leq v \leq \quad \}$

(d) Let M be the surface in part (c). Is the surface integral $\iint_M x \, dS$:

negative zero positive

Circle your answer. **(1 point)**