

1. Suppose  $A$ ,  $B$ , and  $C$  are three distinct points in  $\mathbb{R}^3$  with  $\mathbf{v} = \overrightarrow{AB} = \langle 3, 0, 1 \rangle$  and  $\mathbf{w} = \overrightarrow{AC} = \langle 2, 1, 0 \rangle$ .

(a) Find a normal vector  $\mathbf{n}$  to the plane  $P$  containing  $A, B, C$ . **(2 points)**

$$\mathbf{n} = \langle \quad , \quad , \quad \rangle$$

(b) Find the area of the triangle with vertices  $A, B, C$ . **(2 points)**

$$\text{Area} =$$

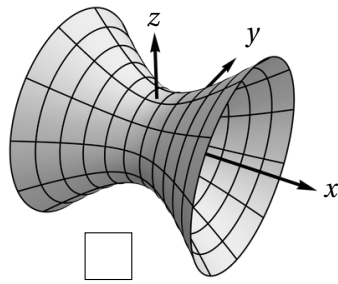
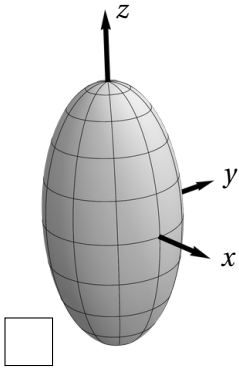
(c) Compute  $\overrightarrow{CB}$ . **(1 point)**

$$\overrightarrow{CB} = \langle \quad , \quad , \quad \rangle$$

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Scratch Space

2. For each quadric surface below, find the corresponding equation from the options at right and write the corresponding letter in the box next to the graph. (2 points each)

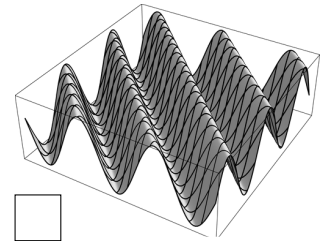
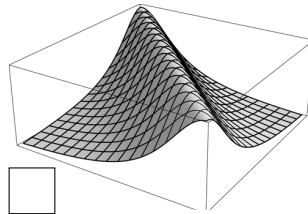
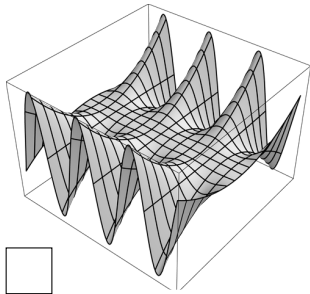
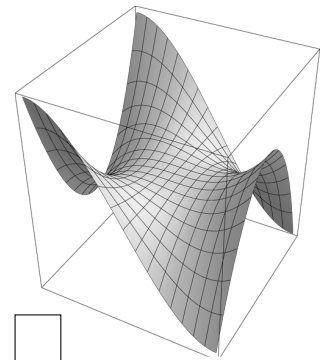
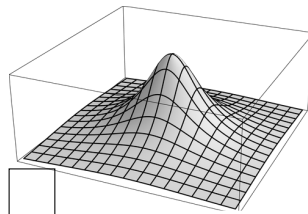
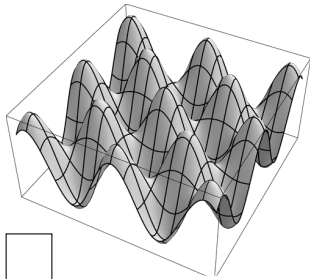


- (A)  $x^2 + y^2 + 4z^2 - 1 = 0$   
 (B)  $x^2 + y^2 - z^2 - 1 = 0$   
 (C)  $4x^2 + 4y^2 + z^2 - 1 = 0$   
 (D)  $x^2 - y^2 - z^2 + 1 = 0$

3. For each function, label its graph from among the options below by writing the corresponding letter in the box next to the graph. (2 points each)

(A)  $\cos(x) \cos(y)$

(B)  $\frac{1}{1 + (x + y)^2}$




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Scratch Space

4. Consider the planes  $V = \{x - y + 2z = 2\}$  and  $W = \{2x - 2y + 4z = 8\}$  in  $\mathbb{R}^3$ .

(a) The two planes  $V$  and  $W$  are parallel. Explain why. **(1 point)**

(b) Find a point  $A$  on  $V$  and a point  $B$  on  $W$ . Compute the distance between  $V$  and  $W$  by using a vector projection involving the vector  $\overrightarrow{AB}$ . **(4 points)**

distance =

5. Let  $f(x, y) = \frac{xy}{x^2 + 2y^2}$  for  $(x, y) \neq 0$ . Does the limit  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exist? If so, what is its value? Justify your answer. **(5 points)**

$$\lim_{(x,y) \rightarrow (0,0)} f(x, y) =$$

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**Scratch Space**

6. Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be the function  $f(x, y) = 2 + \sqrt{x^2 + y^2}$ .

- (a) The function  $f$  is differentiable at  $(1, 0)$ . Find the equation for the tangent plane to its graph at the point  $(1, 0, 3)$ . **(4 points)**

Equation:   $x +$    $y +$    $z =$

- (b) At the point  $(0, 0)$ , the function  $f$  is (circle your answer): **(1 point)**

continuous     differentiable     both     neither

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**Scratch Space**

7. Let  $f(x, y)$  be a function with values and derivatives in the table. Use linear approximation to estimate  $f(3.1, 5.9)$ . (3 points)

| $(x, y)$  | $f(x, y)$ | $\frac{\partial f}{\partial x}(x, y)$ | $\frac{\partial f}{\partial y}(x, y)$ |
|-----------|-----------|---------------------------------------|---------------------------------------|
| $(-1, 3)$ | 0         | 4                                     | 4                                     |
| $(2, 1)$  | 2         | -1                                    | 3                                     |
| $(2, 4)$  | 3         | 7                                     | 7                                     |
| $(3, 6)$  | 1         | -3                                    | 5                                     |

$f(3.1, 5.9) \approx$

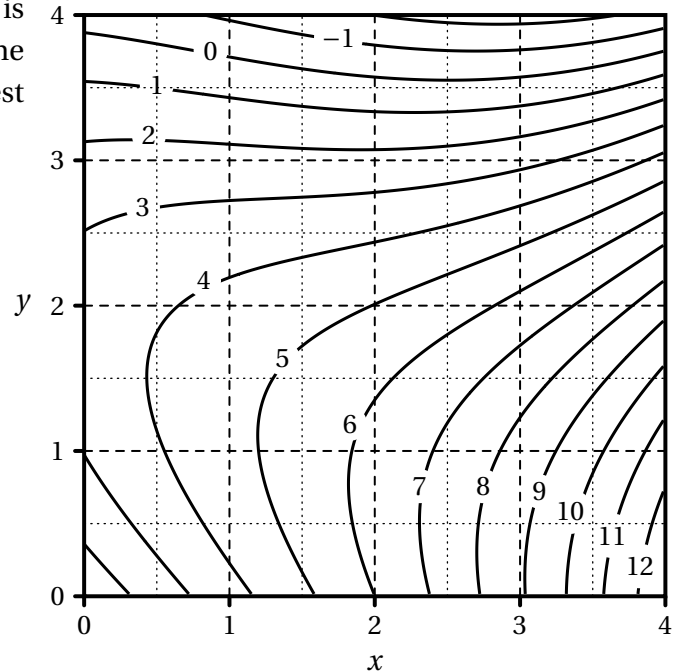
8. The contour map of a differentiable function  $g(x, y)$  is shown at right, where each level curve is labeled by the corresponding value of  $g$ . For each part, circle the best possible answer. (1 point each)

(a)  $\frac{\partial g}{\partial y}(2, 2)$  is

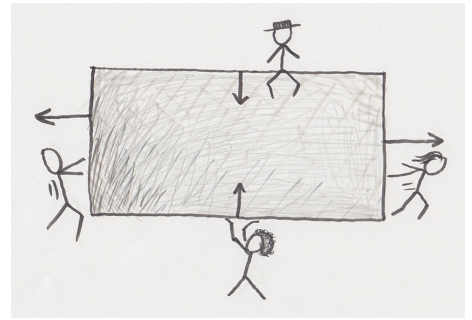
(b)  $\frac{\partial g}{\partial x}(2, 2)$  is

(c)  $\frac{\partial^2 g}{\partial x^2}(2, 2)$  is

(d)  $\frac{\partial^2 g}{\partial y \partial x}(2, 2)$  is



9. A gelatinous rectangle has height  $h = 10m$  and width  $w = 20m$  at time  $t = 0$ . Suppose the rectangle is compressed vertically so the height **decreases** at a rate of  $2m/s$  and is expanded horizontally so that the width **increases** at a rate of  $1m/s$ . Assuming it keeps a rectangular shape throughout, let  $A(t)$  denote its area at time  $t$ . Use the Chain Rule to compute  $\frac{dA}{dt}(0)$ . **(5 points)**



|                      |         |
|----------------------|---------|
| $\frac{dA}{dt}(0) =$ | $m^2/s$ |
|----------------------|---------|

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**Scratch Space**