Practice Exam for Math 241

Instructions: Calculators, books, notes, and suchlike aids to gracious living are not permitted. **Show all your work** as credit will not be given for correct answers without proper justification, except for Problems 2 and 6.

Note: Problem 6(b) and the very last part of 6(a) are based on material from Friday's lecture, so you won't be able to do those yet.

1. Consider the points $A = (0, 0, 2)$, $B = (1, 0, 3)$, and $C = (0, 1, 3)$ in $\mathbb{R}^3$.

   (a) Compute the vectors $\mathbf{v} = \overrightarrow{AB}$ and $\mathbf{w} = \overrightarrow{AC}$. **(2 points)**

   (b) Find a normal vector $\mathbf{n}$ to the plane P containing the points A, B, C. **(3 points)**

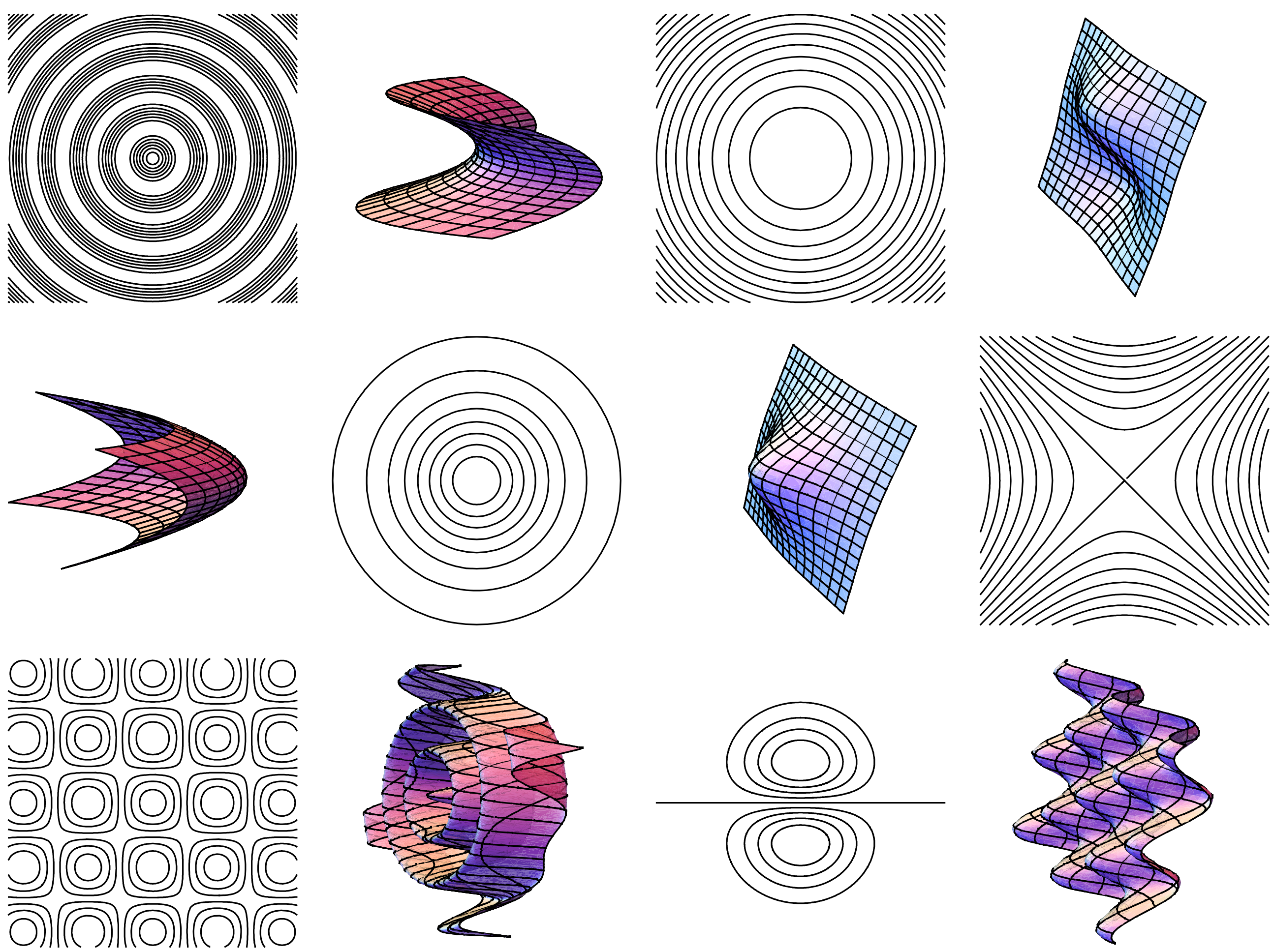
   (c) Find the area of the triangle spanned by A, B, C. **(2 points)**

   (d) Find an equation which describes P. If you can't do (b), take $\mathbf{n} = (1, -2, -1)$. **(1 point)**

   (e) Consider the line L given by the parameterization $\mathbf{r}(t) = (2 + 2t, 3, -1 + 2t)$. Is L parallel to the plane P? Why or why not? **(2 points)**

2. Match the following functions with their graphs and level set diagrams. Here each level set diagram consists of level sets $\{f(\mathbf{x}) = c_i\}$ drawn for evenly spaced c_i. **(9 points)**

   (a) $1/(1 + x^2 + y^2)$ (b) $\cos\sqrt{x^2 + y^2}$ (c) $x^2 - y^2$



3. Consider the function $f(x,y) = \dfrac{y^2}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Compute the following limit, if it exists. **(5 points)**

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

4. Consider the composition of the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ with $x, y\colon \mathbb{R}^2 \to \mathbb{R}$, that is

$$h(s,t) = f\big(x(s,t), y(s,t)\big)$$

Compute $\dfrac{\partial h}{\partial s}(1,2)$ using the chain rule and the table of values at right. **(5 points)**

input	x	y	f	$\frac{\partial x}{\partial s}$	$\frac{\partial y}{\partial s}$	$\frac{\partial f}{\partial x}$	$\frac{\partial f}{\partial y}$
(0,1)	1	1	4	1	2	7	3
(1,1)	1	2	6	1	1	6	2
(1,2)	0	1	5	2	3	5	1
(2,3)	2	3	4	0	1	4	1

5. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = x^2 + \dfrac{x}{y}$.

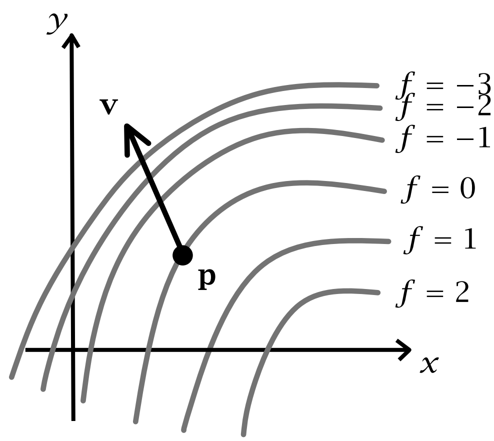
   (a) Compute the partial derivatives f_x, f_y and f_{xy}. **(3 points)**

   (b) Is f differentiable at $(2,1)$? Why or why not? **(2 points)**

   (c) Give the linear approximation of f at the point $(2,1)$: $f(2+\Delta x, 1+\Delta y) \approx$

   (d) Give the equation of the tangent plane to the graph of f at $(2,1,6)$. **(2 points)**

6. The picture below shows some level sets of a function $f\colon \mathbb{R}^2 \to \mathbb{R}$.



   (a) At the point **p** shown, determine the sign of each of the below quantities. **(1 points each)**

   $f(\mathbf{p})$: positive negative 0 $\qquad$ $f_x(\mathbf{p})$: positive negative 0

   $f_y(\mathbf{p})$: positive negative 0 $\qquad$ $f_{xx}(\mathbf{p})$: positive negative 0

   $D_{\mathbf{v}} f(\mathbf{p})$: positive negative 0

   (b) Draw $\nabla f(\mathbf{p})$ on the picture **(1 points)** .

Extra credit problem: Let $E\colon \mathbb{R}^2 \to \mathbb{R}$ be given by $E(x,y) = 3x^2 + xy$. Find a $\delta > 0$ so that $|E(\mathbf{h})| < 0.01$ for all $\mathbf{h} = (x,y)$ with $|\mathbf{h}| < \delta$. Carefully justify why the δ you provide is good enough. **(3 points)**