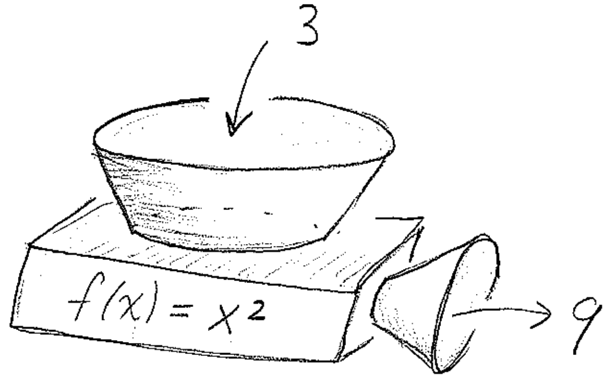
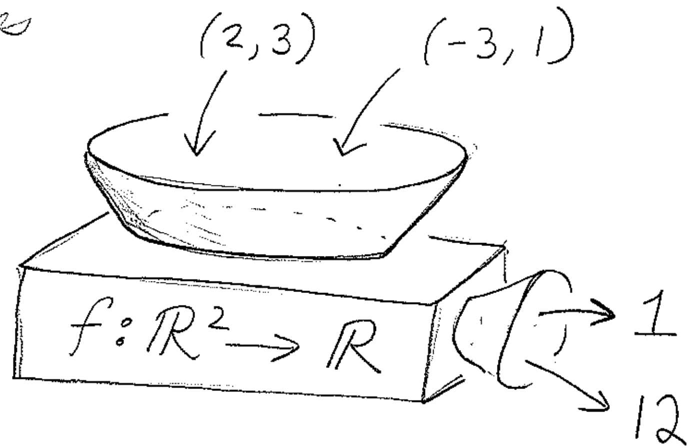
Lecture 5: Functions of Several Variables (14.1)

Function of one variable

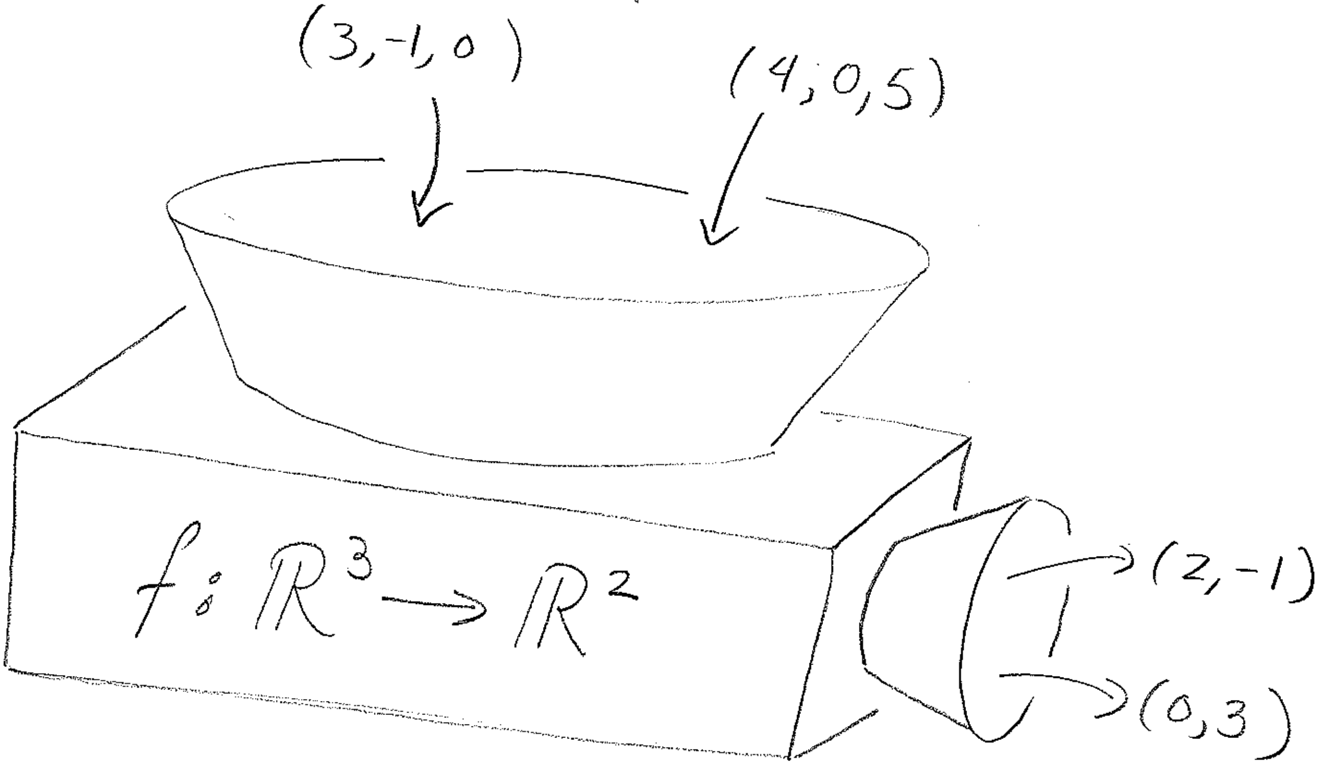


Function of two variables

$$f(x,y) = x^2 - xy$$

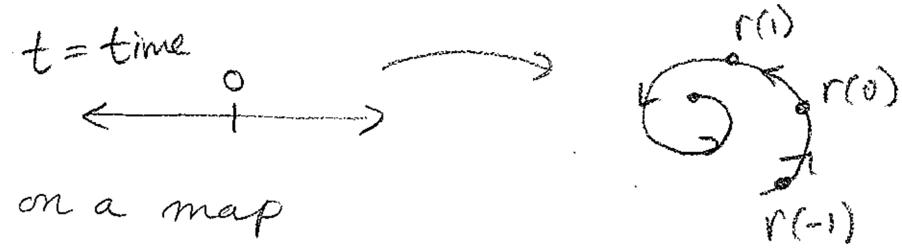


In general will consider $f: \mathbb{R}^n \to \mathbb{R}^m$

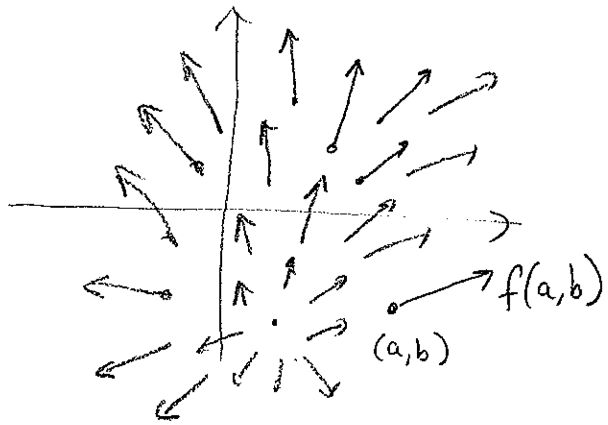


Ex: ① Temperature in this room $T: \mathbb{R}^3 \to \mathbb{R}$

② Parameterized curve in the plane $r: \mathbb{R} \to \mathbb{R}^2$



③ Wind speed / direction on a map



$f: \mathbb{R}^2 \to \mathbb{R}^2$

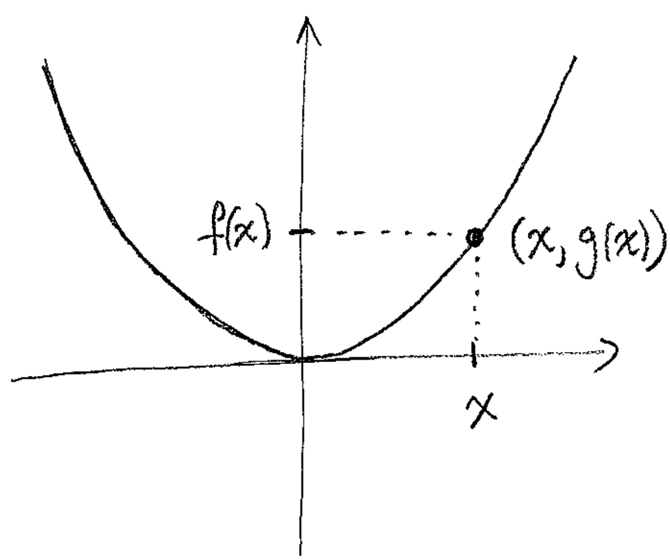
[Eletric field, force due to gravity, etc.]

[Chapter 14 is ①, Chapt 13 is ②, and Chapter 16 includes 3.]

Today: Functions $\mathbb{R}^2 \to \mathbb{R}$

Graphs: One var $f: \mathbb{R} \to \mathbb{R}$

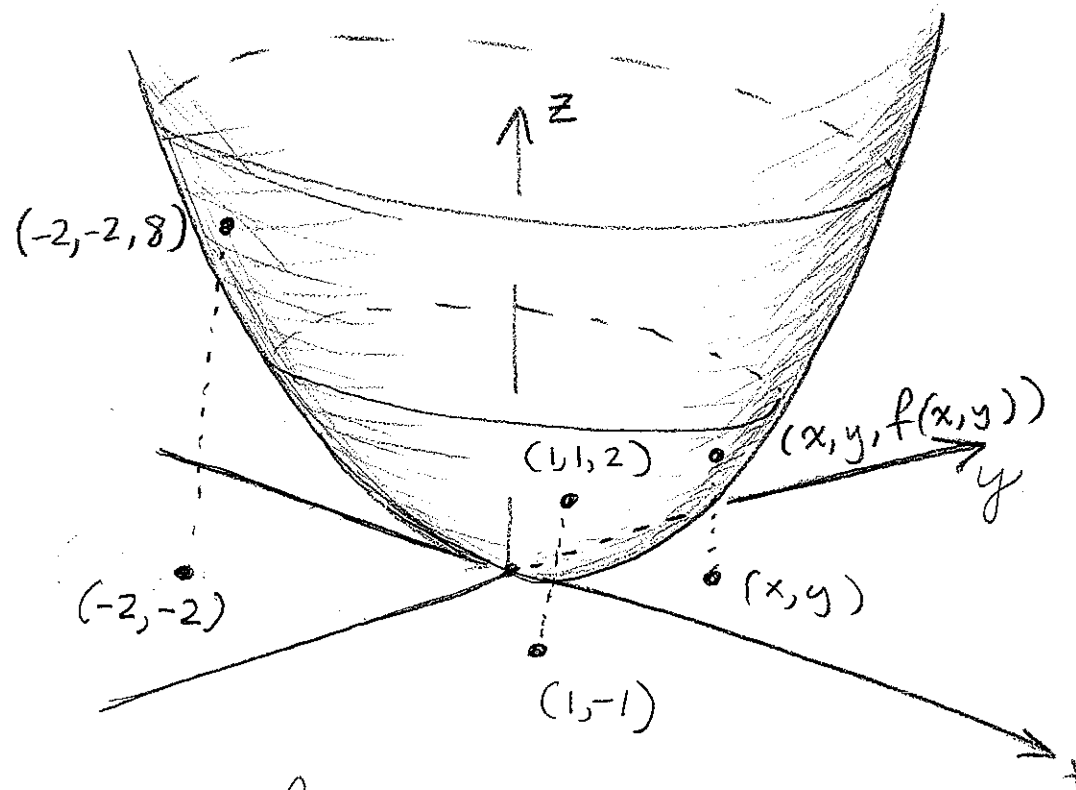
$$f(x) = x^2$$



Consider $f: \mathbb{R}^2 \to \mathbb{R}$

given by $f(x,y) = x^2 + y^2$.

Graph $= \{(x, y, f(x,y))\}$ in $\mathbb{R}^3$

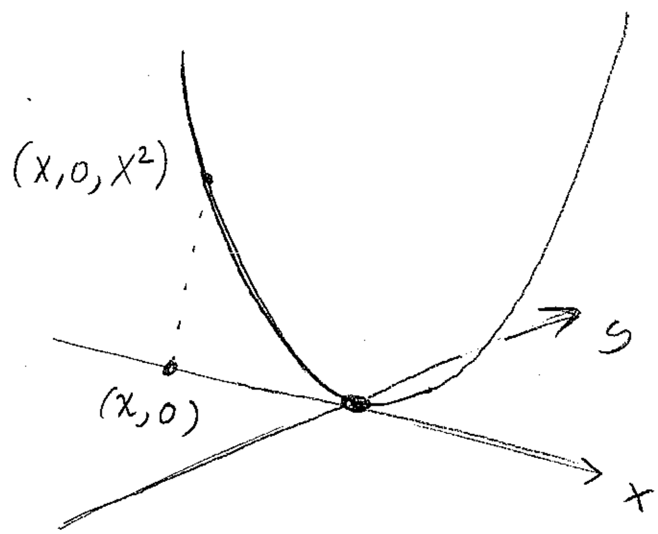


How to figure out:

Intersect with planes:

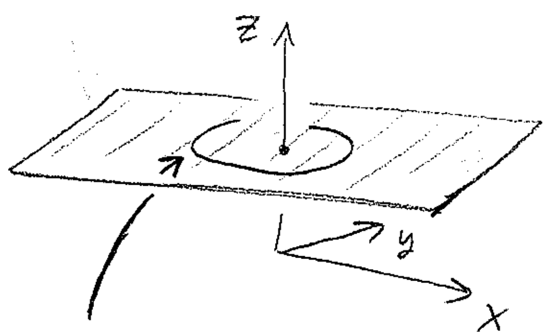
What is over the x-axis?

[or y-axis]



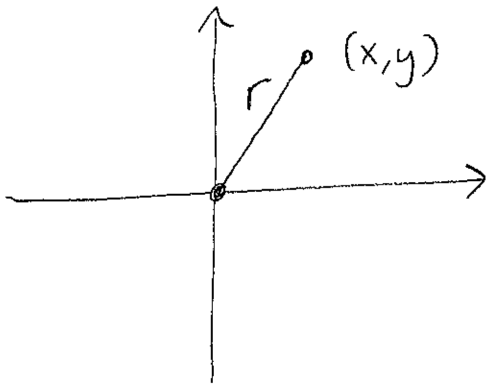
What is the intersection with $\{z = c\}$? Same finding all (x,y) with $f(x,y) = c$ that is

$$x^2+y^2=c$$



Circle of radius $\sqrt{c}$.

<u>Symmetry</u>: $x^2+y^2=r^2$



$\Longrightarrow$ Graph is invariant under rotation around the z-axis.

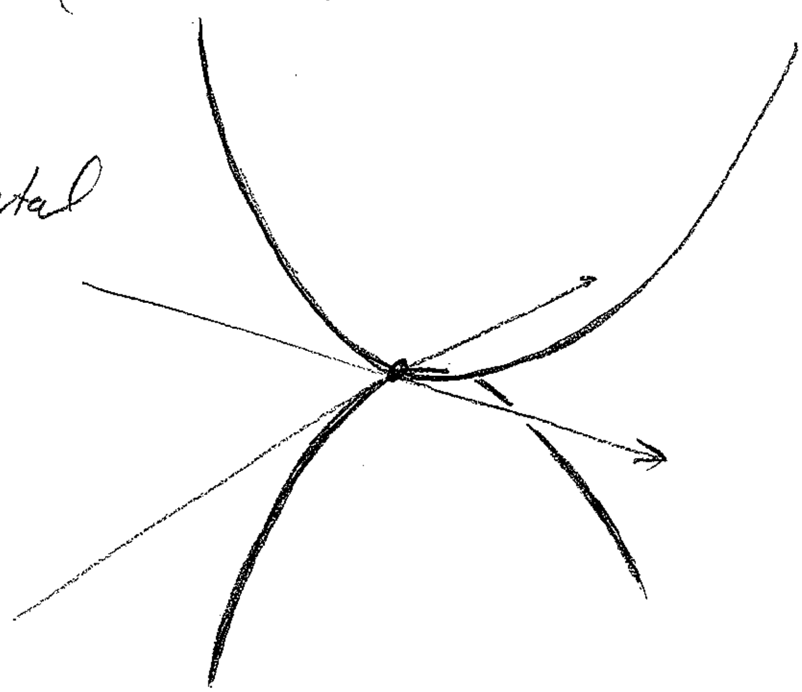
<u>Computer</u>: Can be useful.

<u>Ex</u>: $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = x^2 - y^2$

Over x-axis: $f(x,0) = x^2$ (parabola)

Over y-axis: $f(0,y) = -y^2$ (—— " ——)

Intersections with horizontal planes:



$z=0$: $x^2-y^2=0$

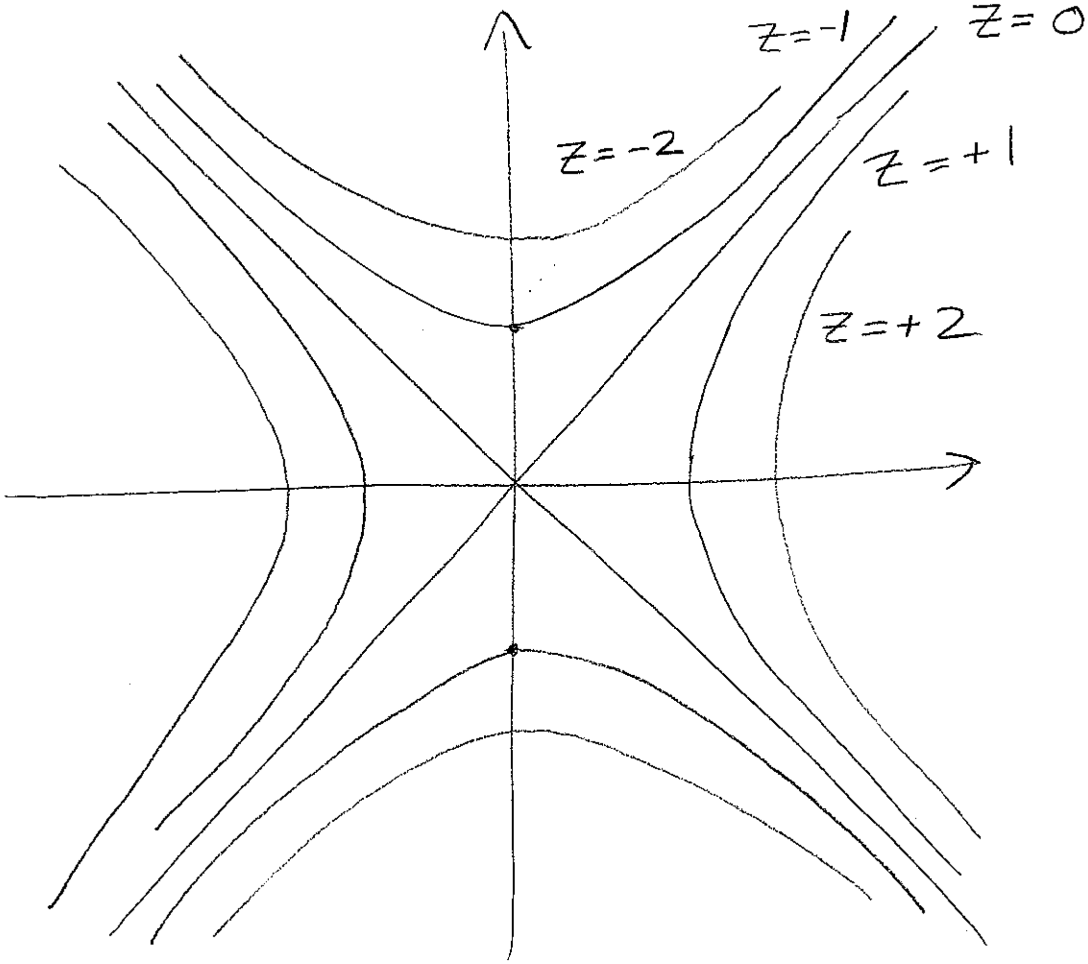
$\Longleftrightarrow x^2=y^2 \Leftrightarrow y=\pm x$

$z=-1$: $x^2-y^2=-1$

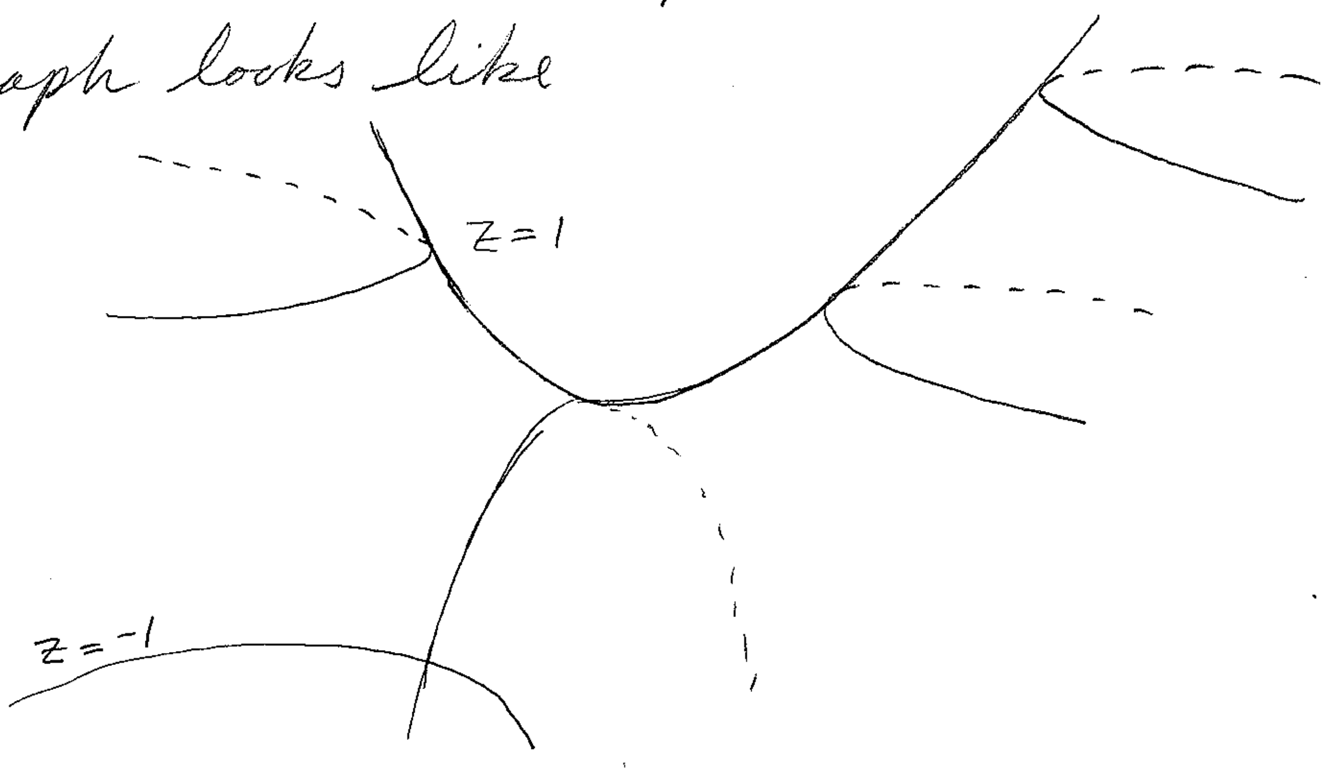
$\Longleftrightarrow y^2=x^2+1$

$z=+1$: $x^2-y^2=+1$

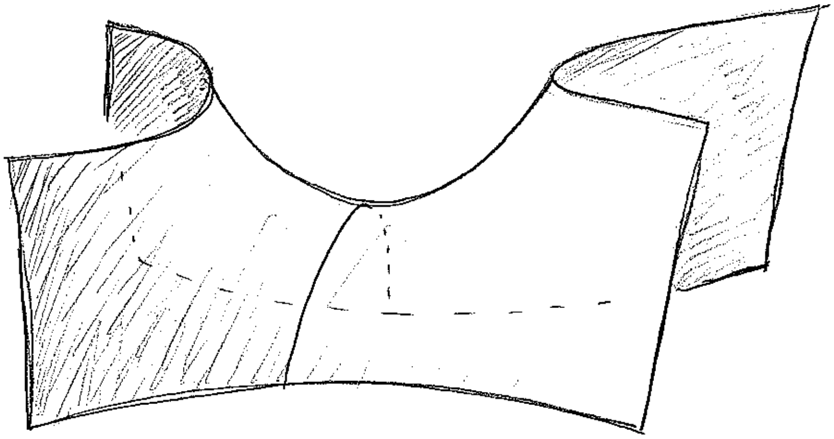
$\Longleftrightarrow x^2=y^2+1$



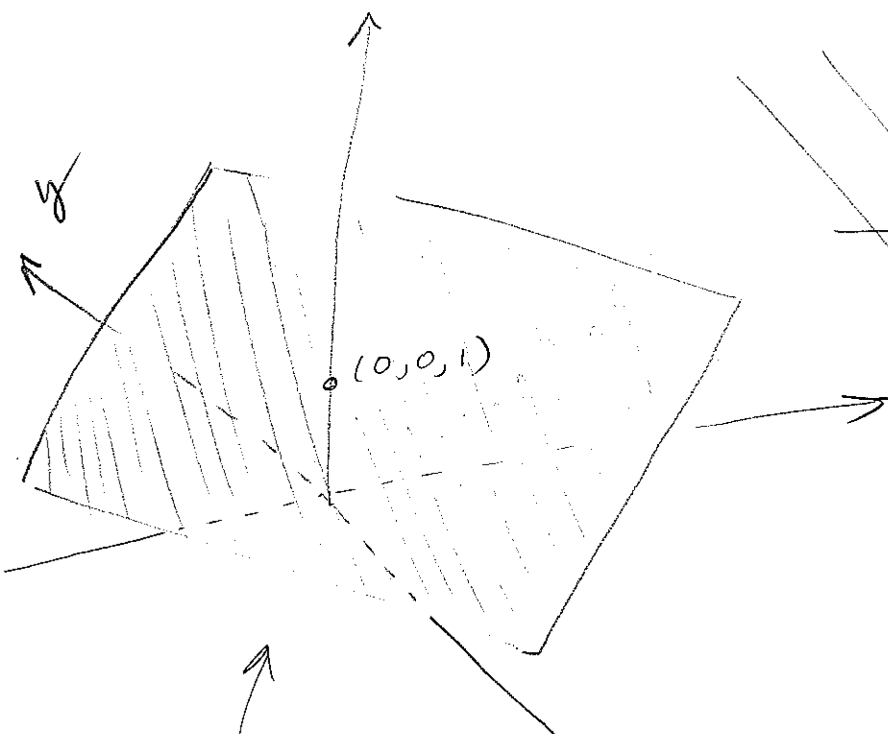
Each of these is a level set, like a contour line on a map. So, the graph looks like

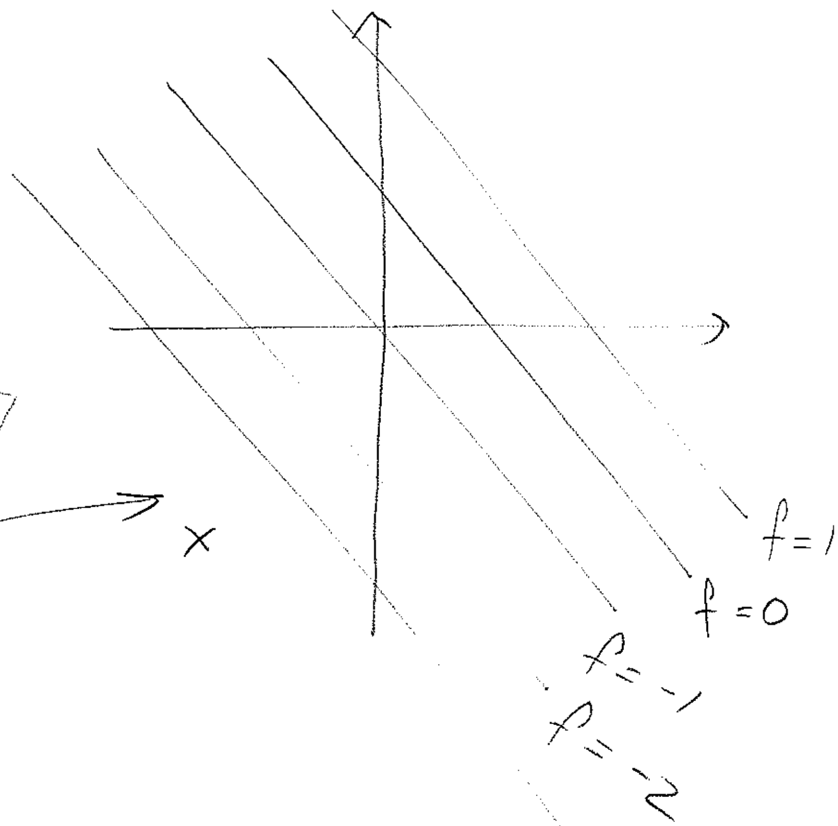


In other words, the graph is a saddle



<u>Ex</u>: $f: \mathbb{R}^2 \to \mathbb{R} \quad f(x,y) = x+y+1$





graph given by $z = f(x,y)$, that is

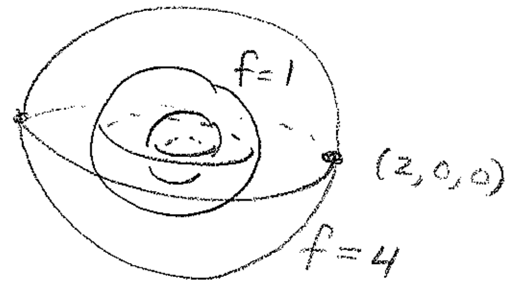
$$z = x+y+1 \iff x+y-z+1 = 0$$

Ex: $f: \mathbb{R}^3 \to \mathbb{R}$ $\quad f(x,y) = x^2+y^2+z^2$

No graph (well, there is one, but its in $\mathbb{R}^4$!)

but we still have level sets.

$f=1 \Leftrightarrow x^2+y^2+z^2=1 \Rightarrow$



Ex: $f: \mathbb{R}^3 \to \mathbb{R}^3$ the "Hopf fibration."

with image contained in .

Level sets are mostly circles....