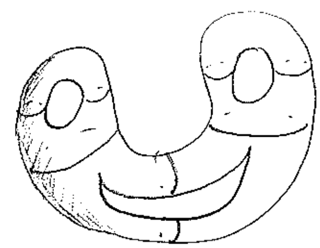
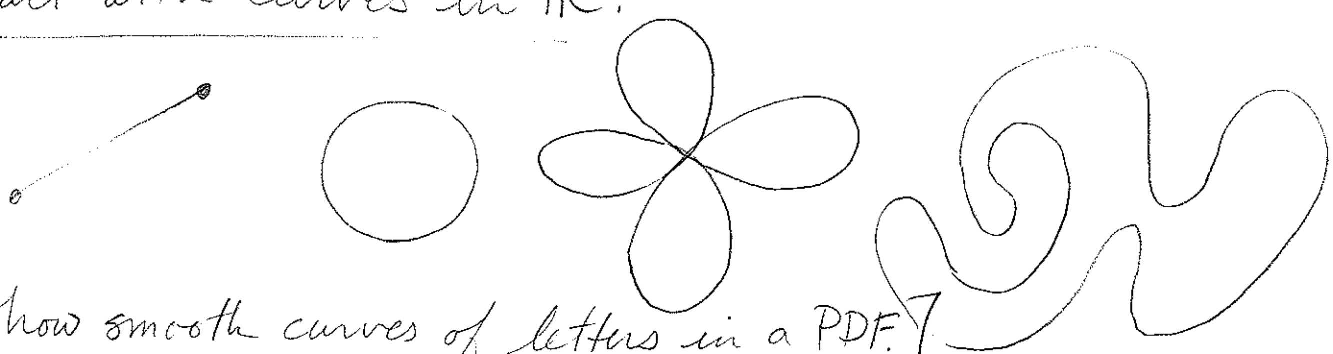
Parameterizing the real world:

Surfaces in Computer Aided Design

Q: How to parameterize the surface of a 787?

or a tea pot? or  ?

Start with curves in $\mathbb{R}^2$:



[Show smooth curves of letters in a PDF.]

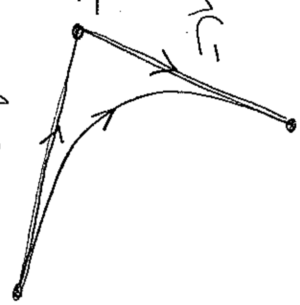
Bézier Curves: [1960s Renault autos.]

Linear:

$$\vec{r}(t) = (1-t)\,\vec{p}_0 + t\,\vec{p}_1$$

"Interpolation."

Quadratic:



Goals: Initially head from p_0 to p_1, and come in to p_2 as if comming from p_1

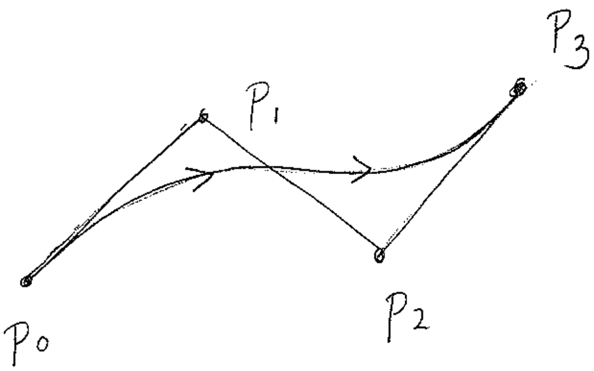
$$\vec{r}(t) = (1-t)\,\vec{r}_0(t) + t\,\vec{r}_1(t)$$
$$= (1-t)\big((1-t)\vec{p}_0 + t\,\vec{p}_1\big) + t\big((1-t)\vec{p}_1 + t\vec{p}_2\big)$$
$$= (1-t)^2\vec{p}_0 + 2t(1-t)\vec{p}_1 + t^2\vec{p}_2$$

Sample Checks: $\vec{r}(0) = \vec{p}_0$

$$\vec{r}\,'(t) = -2(1-t)\vec{p}_0 + 2(1-2t)\vec{p}_1 + 2t^2\vec{p}_2$$

$$\vec{r}\,'(0) = 2(\vec{p}_1 - \vec{p}_0)$$

Cubic:



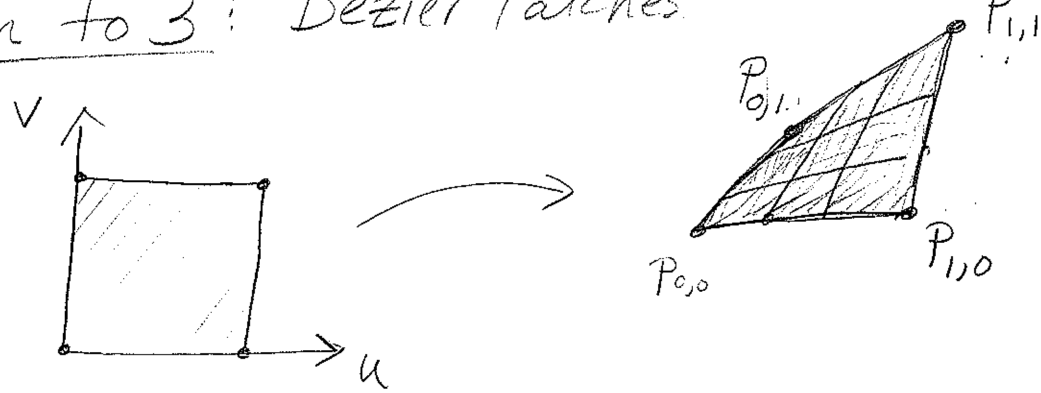
$$\vec{r}(t) = (1-t)\,\vec{r}_{p_0,p_1,p_2}(t) + t\,\vec{r}_{p_1,p_2,p_3}(t)$$

Can do more, but for most things this is enough.

Point: Humans can't see discontinuities in 3rd derivatives.

Note: Transform nicely under linear transformations.

On to 3d Bézier Patches

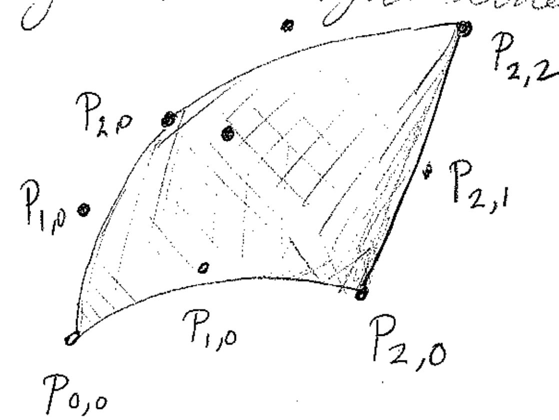


Simplist type:

$$\vec{r}(u,v) = (1-u)(1-v)\,\vec{P}_{0,0} + u(1-v)\,\vec{P}_{1,0} + v(1-u)\,\vec{P}_{0,1} + uv\,\vec{P}_{1,1}$$

Propertres: straight lines go to straight lines.

Biquadratic:



$$\vec{r}(u,v) = (1-u)^2(1-v)^2 P_{0,0} + 2u(1-u)(1-v)^2 P_{1,0} + u^2(1-v)^2 P_{2,0}$$
$$+\ 2(1-u)^2 v(1-v)^2 P_{0,1} + 4u(u-1)v(v-1)\,P_{1,1} + 2u^2 v(1-v) P_{2,1}$$
$$+\ (1-u)^2 v^2 P_{0,2} + 2u(u-1)\,v^2 P_{1,2} + u^2 v^2 P_{2,2}$$

Typical: 16 control points.

Problem: • Matching adjacent patches.

• Issue in computational fluid dynamics.

• Finding intersection of two patches.

• Preserving the topology ...