Tuesday, September 27 $**$ *Taylor series, the* 2^{nd} *derivative test, and changing coordinates.*

1. Consider $f(x, y) = 2\cos x - y^2 + e^{xy}$.

   (a) Show that $(0, 0)$ is a critical point for f.

   (b) Calculate each of f_{xx}, f_{xy}, f_{yy} at $(0, 0)$ and use this to write out the 2^{nd}-order Taylor approximation for f at $(0, 0)$.

   (c) To make sure the next two problems go smoothly, check your answer to (b) with the instructor.

2. Let $g(x, y)$ be the approximation you obtained for $f(x, y)$ near $(0, 0)$ in 1(b).

   (a) It's not clear from the formula whether g, and hence f, has a min, max, or a saddle at $(0, 0)$. Test along several lines until you are convinced you've determined which type it is.

   (b) Check that you're right in (a) using the 2^{nd}-derivative test. The next problem will help explain why this test works.

3. Consider alternate coordinates on $\mathbb{R}^2$ where (u, v) corresponds to $u(1, 1) + v(-1, 1)$.

   (a) Sketch the u- and v-axes, and draw the points whose (u, v)-coordinates are: $(-1, 2)$, $(1, 1)$, $(1, -1)$.

   (b) Give the general formula for the (x, y)-coordinates of a point in terms of u and v. (Like $x = r\cos\theta$ and $y = r\sin\theta$ in polar coordinates.)

   (c) Use (b) to express g as a function of u and v, and expand and simplify the resulting expression.

   (d) Explain why your answer in 3(c) confirms your answer in 2.

   (e) Sketch a few level sets for g. What do the level sets of f look like near $(0, 0)$?

   It turns out that there is always a similar change of coordinates so that the Taylor series of a function f which has a critical point at $(0, 0)$ looks like $f(u, v) \approx f(0, 0) + au^2 + bv^2$.

4. Consider the function $f(x, y) = 3xe^y - x^3 - e^{3y}$.

   (a) Check that f has only one critical point, which is a local maximum.

   (b) Does f have an absolute maxima? Why or why not? Check your answer with the instructor.