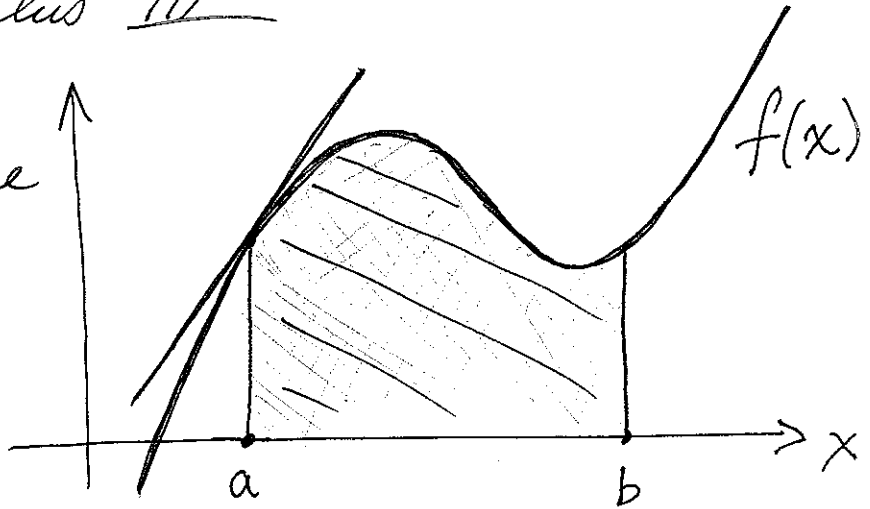


Math 241: Calculus III

(1)

Review in a picture

$f'(a) =$ slope of tangent line

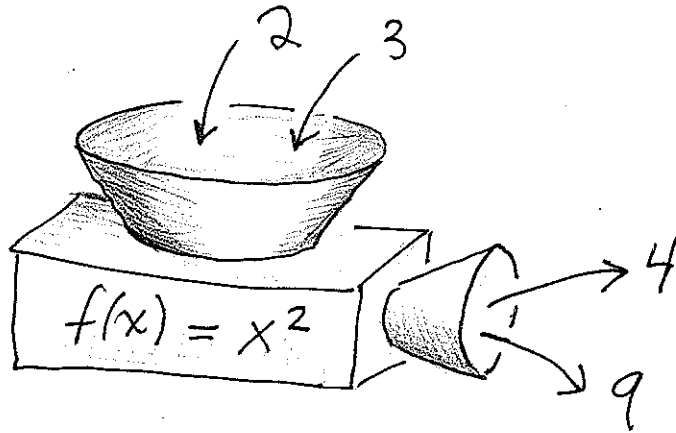


$$\text{Area} = \int_a^b f(x) dx = F(b) - F(a)$$

Fund. Thm of Calc

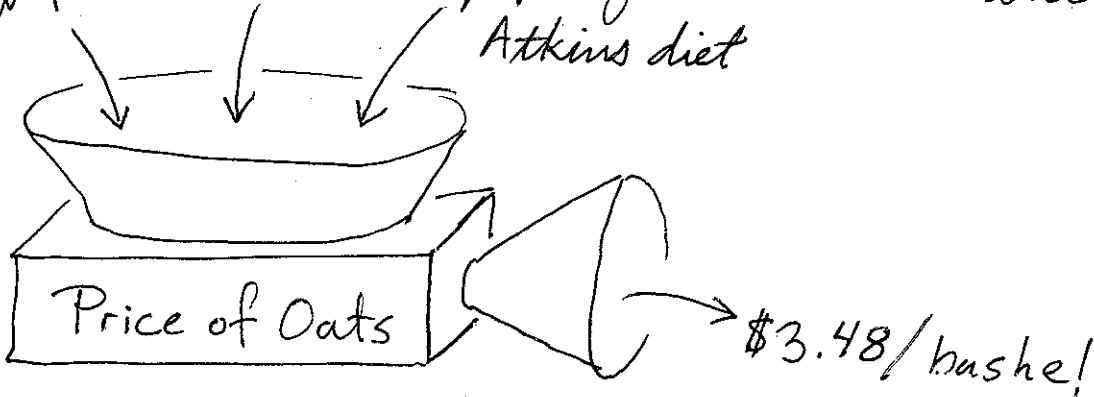
where $F'(x) = f(x)$

Another way to think of $f(x)$:



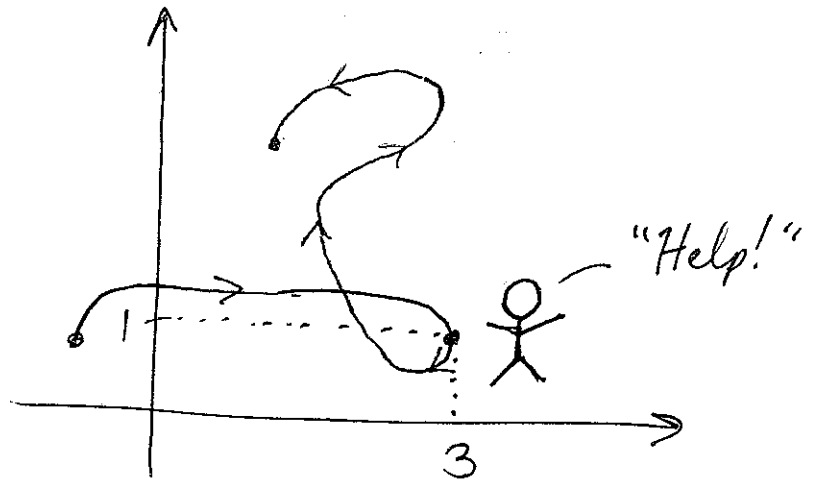
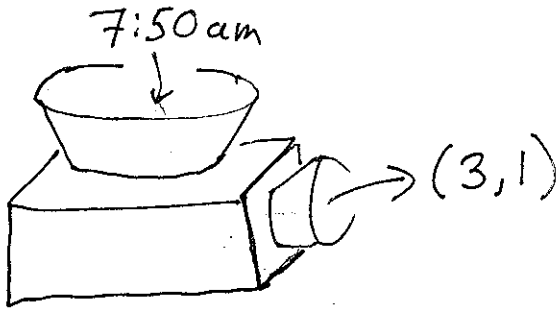
Real life is rarely so simple:

rainfall last frost pop. of Atkins diet



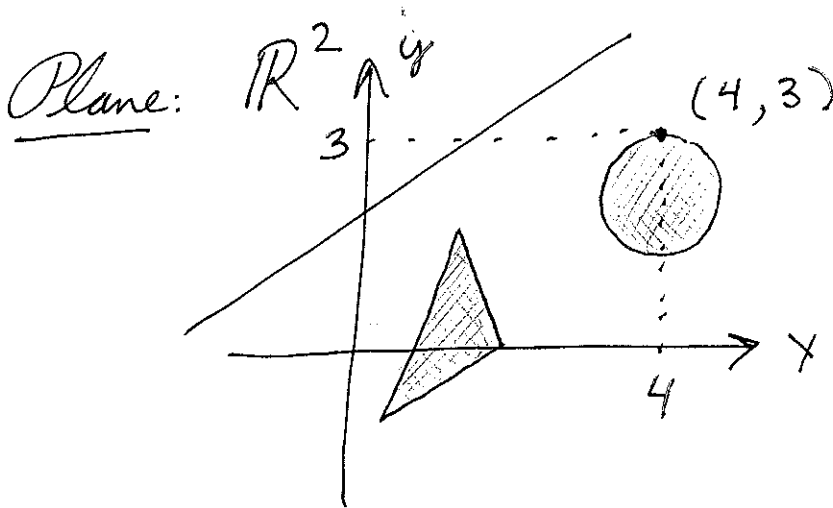
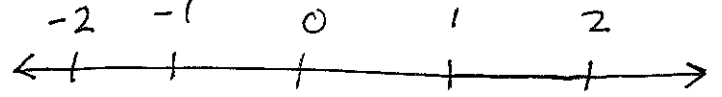
Goal: Understand such functions.

Can have many outputs, too.



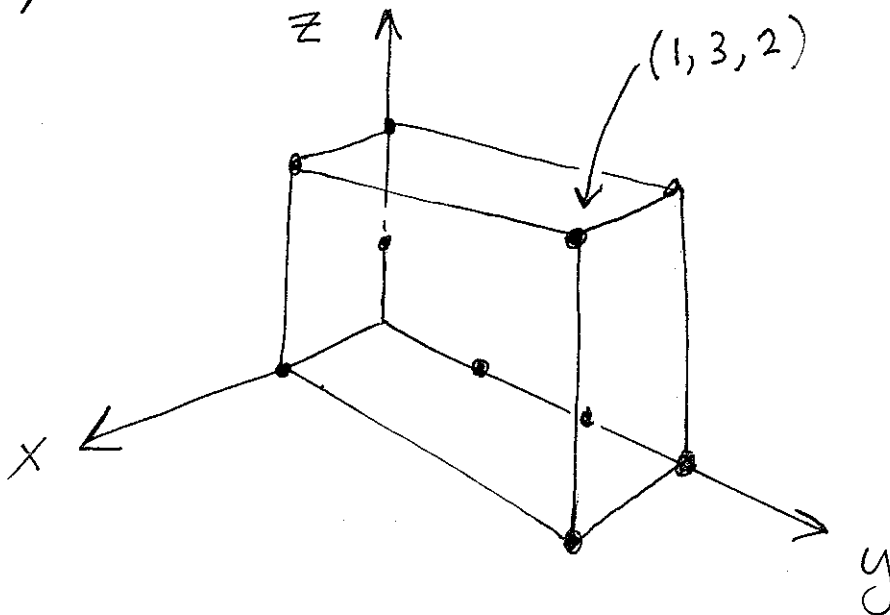
Basic Concept: n -dimensional space. (Section 12.1)

Line: \mathbb{R} = real numbers



\mathbb{R}^2 = pairs (x, y) of real numbers.

3-space: \mathbb{R}^3 [The world we live in]



\mathbb{R}^3 = triples (x, y, z) of #s.

n-space: $\mathbb{R}^n = \text{tuples } (x_1, x_2, x_3, \dots, x_n)$

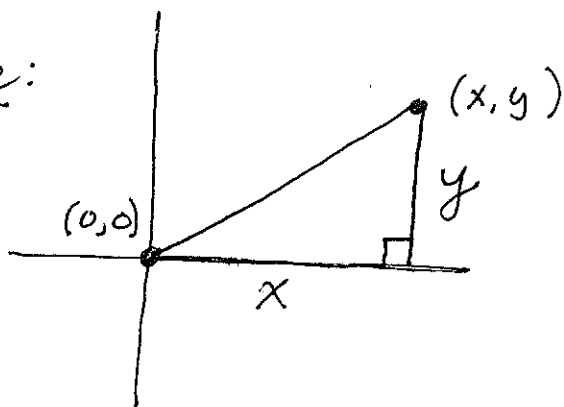
(2)

[In the real world, n might be 10,000...
E.g. NOAA has 1,500 weather monitoring stations.]

[You're familiar with \mathbb{R}^2 and will be with \mathbb{R}^3 ; the power of abstraction in math is that we can use our intuition from the simple to understand the complex.]

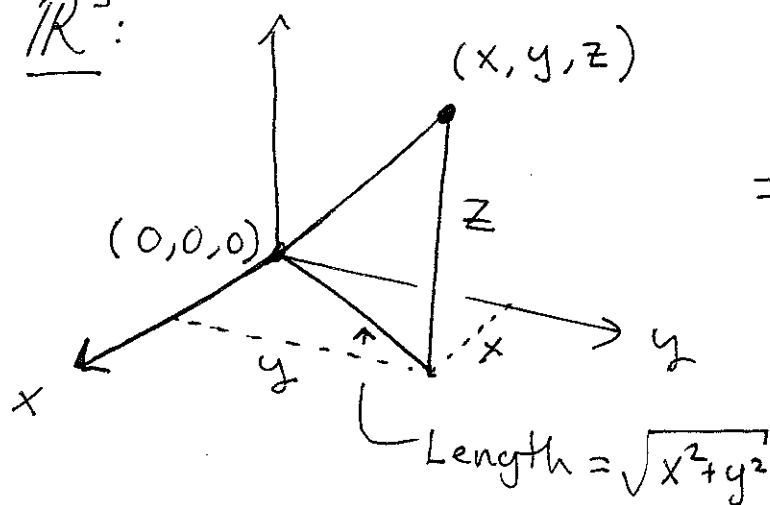
Distance:

\mathbb{R}^2 :



$$\text{dist} = \sqrt{x^2 + y^2}$$

\mathbb{R}^3 :



distance from $(0,0,0)$ to (x,y,z)

$$= \sqrt{(\sqrt{x^2 + y^2})^2 + z^2}$$

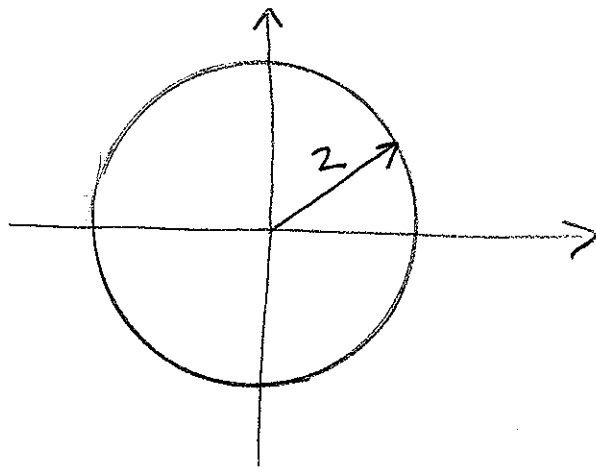
$$= \sqrt{x^2 + y^2 + z^2}$$

\mathbb{R}^n : Distance between

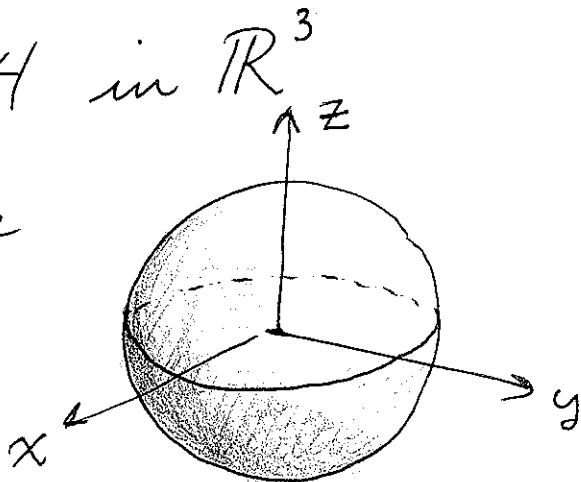
$P(x_1, x_2, \dots, x_n)$ and $Q(y_1, y_2, \dots, y_n)$

$$\text{is } |PQ| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}$$

Ex: $x^2 + y^2 = 4$ in \mathbb{R}^2
= circle



$x^2 + y^2 + z^2 = 4$ in \mathbb{R}^3
= sphere



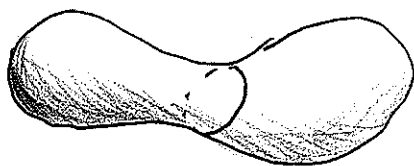
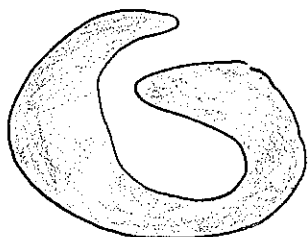
$x^2 + y^2 + z^2 + w^2 = 4$ in $\mathbb{R}^4 = ???$

Course Outline

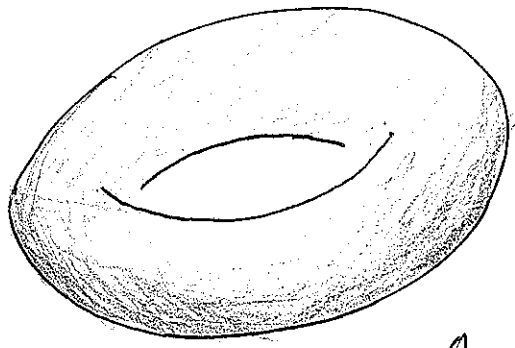
- Vectors and geometry of \mathbb{R}^n . [planes, lines, dot and cross products.]
- Functions of several variables

Differentiation \implies Optimization (min/max)

Integration: Areas, volumes, averages over multidimension objects

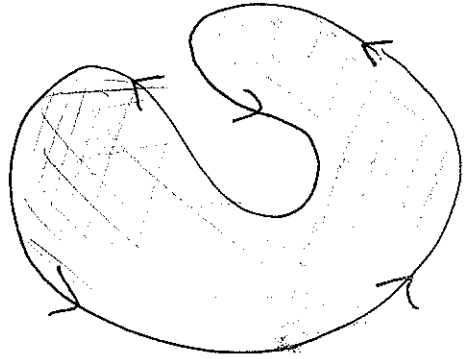


- Curves and surfaces in \mathbb{R}^3



• Relationships between these, generalizing the Fund. Thm of Calc.

Ex: Figuring out the area by integrating around the boundary.



[course highlight, connected to E+M.]

Go over syllabus.

