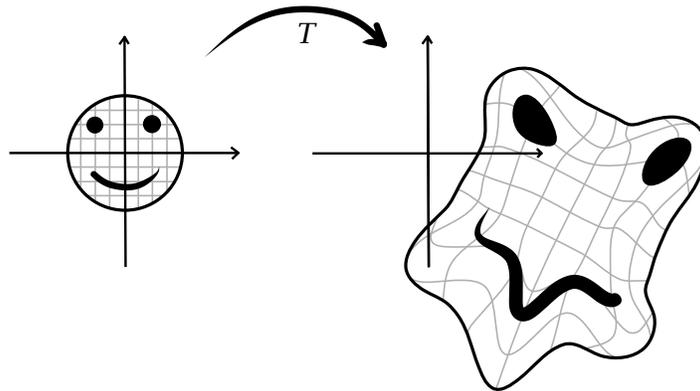


Discussion – Thursday, October 28th: Transformations of \mathbb{R}^2 .

Purpose: In class, we've seen several different coordinate systems on \mathbb{R}^2 and \mathbb{R}^3 beyond the usual rectangular ones: polar, cylindrical, and spherical. The first two lectures of next week will cover the crucial technique of simplifying hard integrals using a change of coordinates (Section 15.9). The point of this worksheet is to familiarize you with some basic concepts and examples for this process.

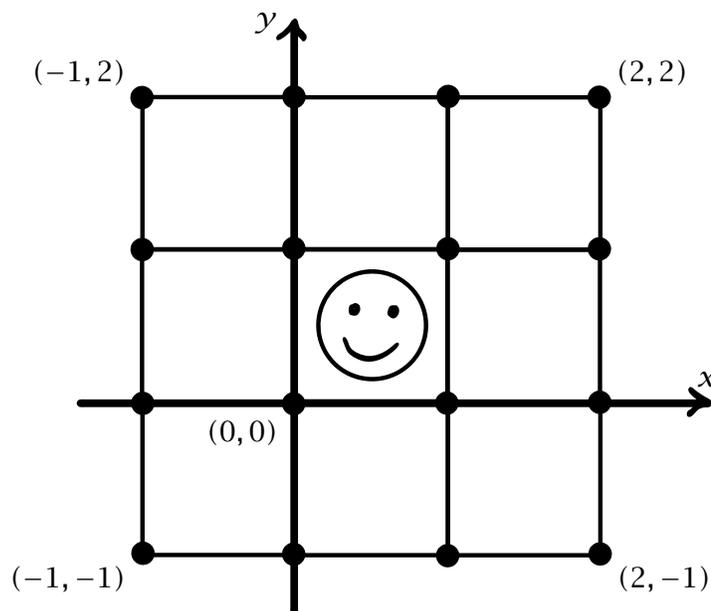
Starting point: Here we consider a variety of transformations $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$. Previously, we have used such functions to describe vector fields on the plane, but we can also use them to describe ways of distorting the plane:



1. Consider the transformation $T(x, y) = (x - 2y, x + 2y)$.

(a) Compute the image under T of each vertex in the below grid and make a careful plot of them, which should be fairly large as you will add to it later.

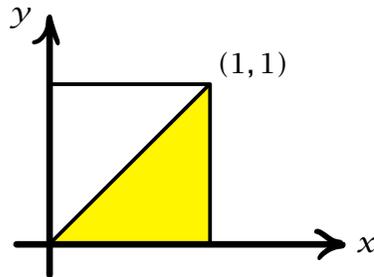
To speed this up, divide the task up among all members of the group.



- (b) For each pair A and B of vertices of the grid joined by a line, add the line segment joining $T(A)$ to $T(B)$ to your plot. This gives a rough picture of what T is doing. Check your answer with the instructor.
- (c) What is the image of the x -axis under T ? The y -axis?
- (d) Consider the line L given by $x + y = 1$. What is the image of L under T ? Is it a circle, an ellipse, a hyperbole, or something else?
Hint: First, parameterize L by $\mathbf{r}: \mathbb{R} \rightarrow \mathbb{R}^2$ and then consider $\mathbf{f}(t) = T(\mathbf{r}(t))$.
- (e) Consider the circle C given by $x^2 + y^2 = 1$. What is the image of C under T ?
- (f) Add $T(L)$, $T(C)$ and $T(\text{☺})$ to your picture. Check your answer with the instructor.

Note: The transformation T is a particularly simple sort called a *linear transformation*.

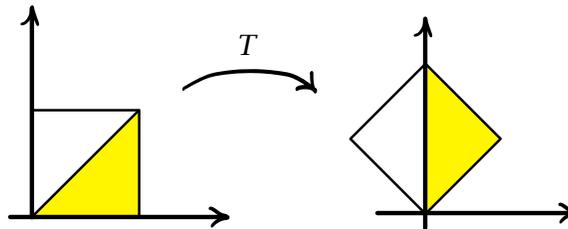
2. Consider the transformation $T(x, y) = (y, x(1 + y^2))$. Draw the image of the picture below under T .



Hint: Parameterize each of the 5 line segments and proceed as in 1(d). To speed things, divide up the task.

Check your answer with the instructor.

3. In this problem, you'll construct a transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ which rotates counter-clockwise about the origin by $\pi/4$, as shown below.



- (a) Give a formula for T in terms of polar coordinates. That is, how does rotation affect r and θ ?
- (b) Write down T in terms of the usual rectangular (x, y) coordinates. Hint: first convert into polar, apply part (a) and then convert back into rectangular coordinates.

Check your answer with the instructor.