Discussion – Tuesday, October 26th: Introduction to multiple integrals

1. Consider $\int_{-1}^{1}\int_{0}^{1} \left(6-2x-3y\right) dy\, dx$.

   (a) Evaluate this iterated integral.

   (b) Reverse the order of integration and show that you get the same answer. (Hint: When you change to $dx\, dy$, be sure to also change the bounds of integration.)

   (c) Show that the integrand is non-negative over the region $R = [-1,1] \times [0,1]$ in its domain. Explain how this is consistent with your answers in (a) and (b).

   (d) Sketch the solid in $\mathbb{R}^3$ whose volume is given by the iterated integral.

2. Consider the region in $\mathbb{R}^2$ bounded by the line $y = 2x$ and the parabola $y = x^2$.

   (a) Sketch the region R.

   (b) Using Calc I methods, set up an integral of the form $\int_a^b \left(f(x) - g(x)\right) dx$ that calculates the area of R.

   (c) Consider the iterated integral $\int_0^2 \int_{x^2}^{2x} dy\, dx$. Evaluate this integral and explain why this double integral also calculates the area of R.

3. Consider the region bounded by the curves determined by $-2x + y^2 = 6$ and $-x + y = -1$.

   (a) Sketch the region R in the plane.

   (b) Set up and evaluate an integral of the form $\iint_R dA$ that calculates the area of R.

   (c) Calculate the average of $f(x, y) = x + y$ on R.

4. Consider the region R which lies above the x-axis and between the circles of radius 1 and 2 centered at $(0,0)$. (This the region from Monday's lecture.) *Without using polar coordinates,* evaluate

   $$\iint_R y\, dA$$

   Check your answer against what Nathan got in class.

   Hint: You'll have to break R into several simple (Type I and II) regions. If you use the symmetry of the situation, you can get by doing just two double integrals.