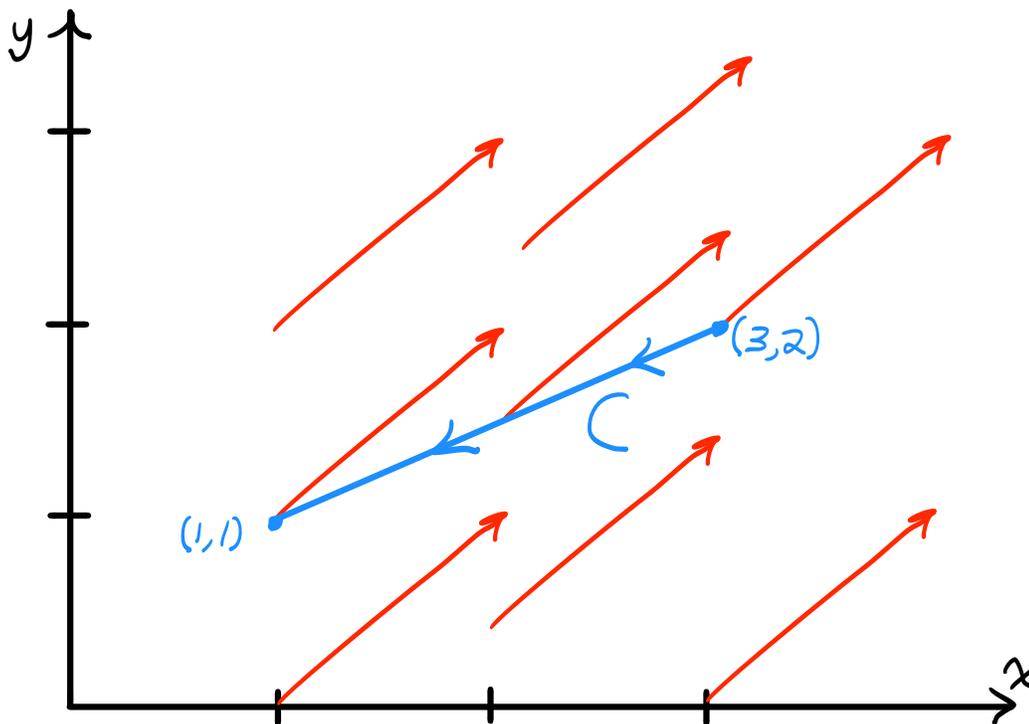


**Discussion — Tuesday, October 12th**

**Subject:** Integrating vector fields.

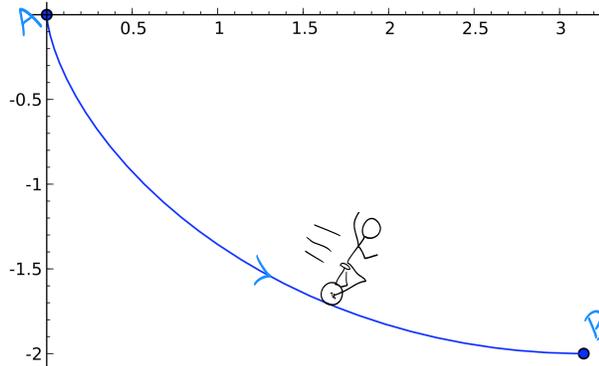
1. Consider the curve  $C$  and vector field  $\mathbf{F}$  shown below.



- (a) Without parameterizing  $C$ , evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$ . (Hint: Use one of the reformulations of  $\int_C \mathbf{F} \cdot d\mathbf{r}$  from Monday's lecture.)
- (b) Find a parameterization of  $C$  and use it to check your answer in (a) by computing  $\int_C \mathbf{F} \cdot d\mathbf{r}$  explicitly.
2. Consider the vector field  $\mathbf{F} = (y, 0)$  on  $\mathbb{R}^2$ .
- (a) Draw a sketch of  $\mathbf{F}$  on the region where  $-2 \leq x \leq 2$  and  $-2 \leq y \leq 2$ . Check your answer with the instructor.
- (b) Consider the following two curves which *start* at  $A = (-2, 0)$  and *end* at  $B = (2, 0)$ , namely the line segment  $C_1$  and upper semicircle  $C_2$ .  
Add these curves to your sketch, and compute both  $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$  and  $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ . Check your answers with the instructor.
- (c) Based on your answer in (b), could  $\mathbf{F}$  be  $\nabla f$  for some  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ ? Explain why or why not.

**Note:** More problems on the back.

3. Consider the points  $A = (0, 0)$  and  $B = (\pi, -2)$ . Suppose an object of mass  $m$  moves from  $A$  to  $B$  and experiences the constant force  $\mathbf{F} = -mg\mathbf{j}$ , where  $g$  is the gravitational constant.
- (a) If the object follows the straight line from  $A$  to  $B$ , calculate the work  $W$  done by gravity using the formula from the first week of class.
- (b) Now suppose the object follows half of an inverted cycloid  $C$  as shown below. Explicitly parameterize  $C$  and use that to calculate the work done via a line integral.



- (c) Find a function  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  so that  $\nabla f = \mathbf{F}$ . Use the Fundamental Theorem of Line Integrals to check your answers for (a) and (b). Have you seen the quantity  $-f$  anywhere before? If so, what was its name?

4. If you get this far, work #48 from Section 16.2:

48. Experiments show that a steady current  $I$  in a long wire produces a magnetic field  $\mathbf{B}$  that is tangent to any circle that lies in the plane perpendicular to the wire and whose center is the axis of the wire (as in the figure). *Ampère's Law* relates the electric current to its magnetic effects and states that

$$\int_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I$$

where  $I$  is the net current that passes through any surface bounded by a closed curve  $C$ , and  $\mu_0$  is a constant called the permeability of free space. By taking  $C$  to be a circle with radius  $r$ , show that the magnitude  $B = |\mathbf{B}|$  of the magnetic field at a distance  $r$  from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r}$$

