

Discussion — Tuesday, September 28th

Subject: Taylor series, the second derivative test, and changing coordinates.

1. Consider $f(x, y) = 5(\sin^2(x) + y^2) - 3e^{-(x+1)y} + 3e^{-y} + 1$.
 - (a) Show that $(0, 0)$ is a critical point for f .
 - (b) Calculate each of f_{xx} , f_{xy} , f_{yy} at $(0, 0)$ and use this to write out the 2nd-order Taylor approximation for f at $(0, 0)$.
 - (c) To make sure the next two problems go smoothly, check your answer to (b) with the instructor.
2. Let $g(x, y)$ be the approximation you obtained for $f(x, y)$ near $(0, 0)$ in 1(b).
 - (a) It's not clear from the formula whether g , and hence f , has a min, max, or a saddle at $(0, 0)$. Test along several lines until you are convinced you've determined which type it is.
 - (b) Check that you're right in (a) using the 2nd-derivative test. The next problem will help explain why this test works.
3. Consider alternate coordinates on \mathbb{R}^2 where (u, v) corresponds to $u(1, 1) + v(-1, 1)$.
 - (a) Sketch the u - and v -axes, and draw the points whose (u, v) -coordinates are: $(-1, 2)$, $(1, 1)$, $(1, -1)$.
 - (b) Give the general formula for the (x, y) -coordinates of a point in terms of u and v . (Like $x = r \cos \theta$ and $y = r \sin \theta$ in polar coordinates.)
 - (c) Use (b) to express g as a function of u and v , and expand and simplify the resulting expression.
 - (d) Explain why your answer in 3(c) confirms your answer in 2.
 - (e) Sketch a few level sets for g . What do the level sets of f look like near $(0, 0)$?

It turns out that there is always a similar change of coordinates so that the Taylor series of a function f which has a critical point at $(0, 0)$ looks like $f(u, v) = f(0, 0) + au^2 + bv^2$.

4. Consider the function $f(x, y) = 3xe^y - x^3 - e^{3y}$.
 - (a) Check that f has only one critical point, which is a local maximum.
 - (b) Does f have a global maxima? Why or why not? Check your answer with the instructor.
5. For the function shown on the back of the sheet, use the level curves to find the locations and types (min/max/saddle) for all the critical points of the function:

$$f(x, y) = 3x - x^3 - 2y^2 + y^4$$

Use the formula for f and the 2nd-derivative test to check your answer.

