Discussion – Tuesday, September 14th

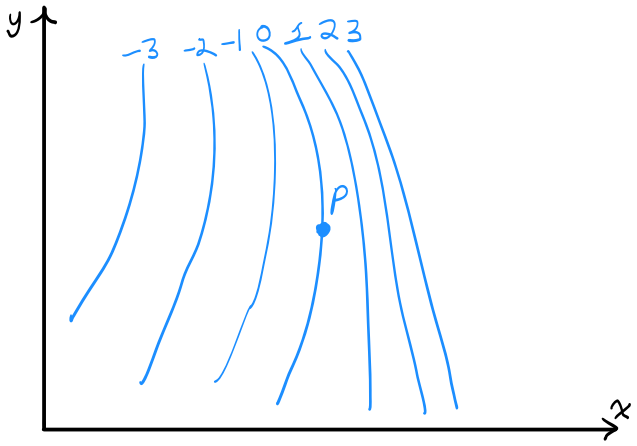
1. Consider $f(x,y) = xy^2e^{x^2}$. Compute the following:

$$\frac{\partial f}{\partial x} \qquad \frac{\partial f}{\partial y} \qquad \frac{\partial^2 f}{\partial x \partial y} \qquad \frac{\partial^2 f}{\partial y \partial x}$$

   What is the relationship between your answers for the last two? This is an instance of Clairaut's Theorem and holds for most functions.

2. Shown are some level curves for the function $f\colon \mathbb{R}^2 \to \mathbb{R}$. Determine whether the following partial derivatives are positive, negative, or zero at the point P:

$$f_x \qquad f_y \qquad f_{xx} \qquad f_{yy} \qquad f_{xy} = f_{yx}$$



3. The wind-chill index $W = f(T,v)$ is the perceived temperature when the actual temperature is T and the wind speed is v. Here is a table of values for W.

Wind speed (km/h)

Actual temperature (°C) T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

   (a) Use the table to estimate $\frac{\partial f}{\partial T}$ and $\frac{\partial f}{\partial v}$ at $(T,v) = (-20,40)$.

   (b) Use your answer in (a) to write down the linear approximation to f at $(-20,40)$.

   (c) Use your answer in (b) to approximate $f(-22,45)$.

4. Consider $f(x,y) = \sqrt{1-x^2-y^2}$.

   (a) What is the domain of f? That is, for which (x,y) does the function make sense?

   (b) Describe geometrically the surface which is the graph of f.

   (c) Find the tangent plane to the graph at $(x,y) = (1/2, 1/2)$.

   (d) Consider the vector $\mathbf{v} = \left(1/2, 1/2, 1/\sqrt{2}\right)$ which goes from $\mathbf{0}$ to the point on the graph where we just found the tangent plane. What is the angle between $\mathbf{v}$ and a normal vector to the tangent plane?

5. Consider $f(x,y) = \sqrt[3]{x^3+y^3}$.

   (a) Compute $f_x(0,0)$. Note: this partial derivative exists.

   (b) Is f differentiable at $(0,0)$?