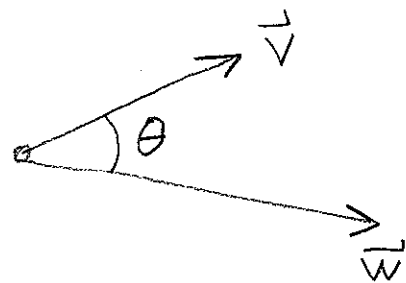


Lecture 3: Dot product (12.3) and lines and planes in \mathbb{R}^3 (12.5)

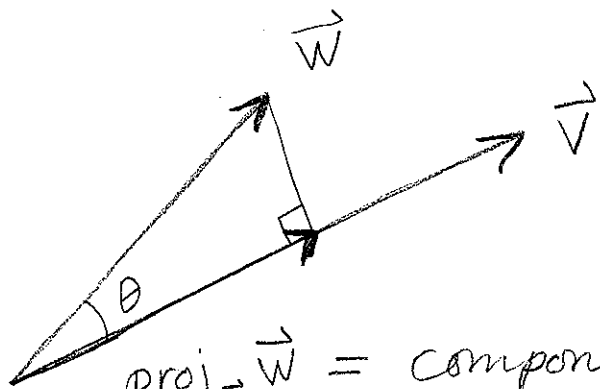
Last time: $\vec{v} = (v_1, v_2, v_3)$, $\vec{w} = (w_1, w_2, w_3)$

$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

Key: $\vec{v} \cdot \vec{w} = |\vec{v}| |\vec{w}| \cos \theta$



Projection:



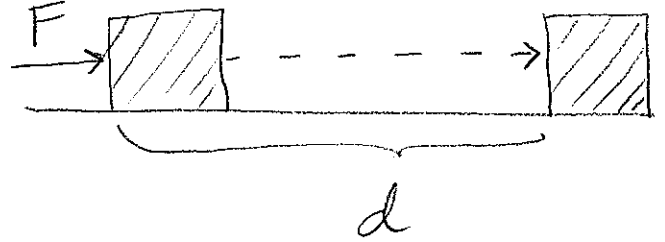
$\text{proj}_{\vec{v}} \vec{w} =$ component of \vec{w} along \vec{v}
 $=$ scalar mult of \vec{v}
closest to \vec{w} .

Note: $|\text{proj}_{\vec{v}} \vec{w}| = |\vec{w}| \cos \theta$

So: $\text{proj}_{\vec{v}} \vec{w} = |\vec{w}| \cos \theta$ (unit vector pointing in same dir as $\vec{v} = \frac{\vec{v}}{|\vec{v}|}$)

$$= \frac{|\vec{v}| |\vec{w}| \cos \theta}{|\vec{v}|^2} \vec{v} = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}|^2} \vec{v}$$

Work = (force) \times (distance)

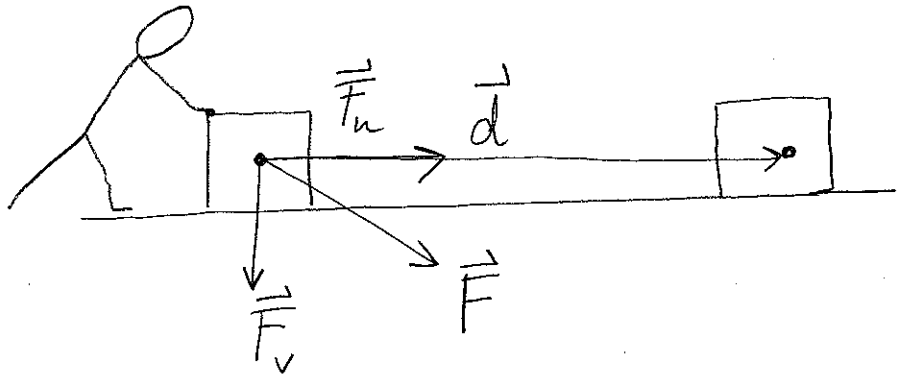


$$W = |\vec{F}_n| |\vec{d}|$$

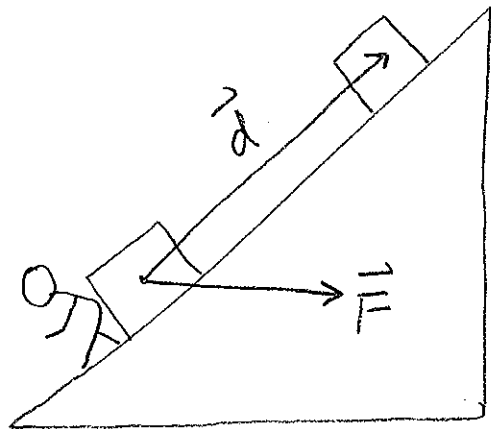
$$= |\text{proj}_{\vec{d}} \vec{F}| |\vec{d}|$$

$$= \left| \frac{\vec{F} \cdot \vec{d}}{|\vec{d}|^2} \vec{d} \right| |\vec{d}|$$

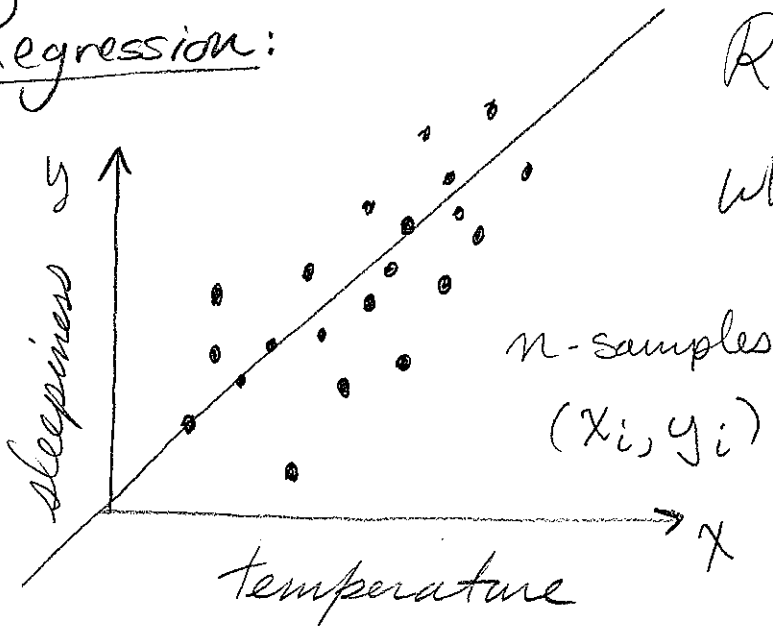
$$= \vec{F} \cdot \vec{d}$$



$$W = \vec{F} \cdot \vec{d}$$



Regression:



Roughly $y = cx$.
What is c ?

$$\vec{x} = (x_1, x_2, x_3, \dots, x_n)$$

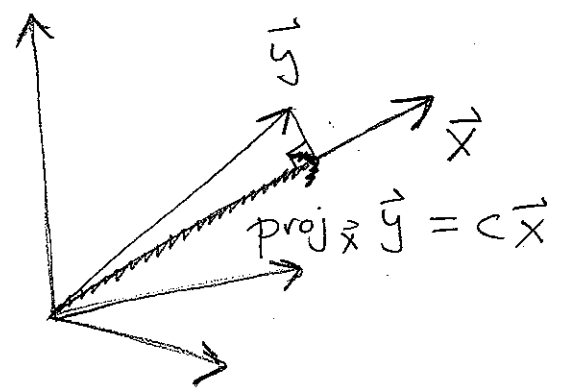
$$\vec{y} = (y_1, y_2, y_3, \dots, y_n)$$

cf $y_i = c x_i$ for each i , then

$$\vec{y} = c \vec{x} \quad \mathbb{R}^n$$

So "best fit" is

$$c = \frac{\vec{x} \cdot \vec{y}}{|\vec{x}|^2}$$



which minimizes $|\vec{y} - c \vec{x}|$ [cf. general

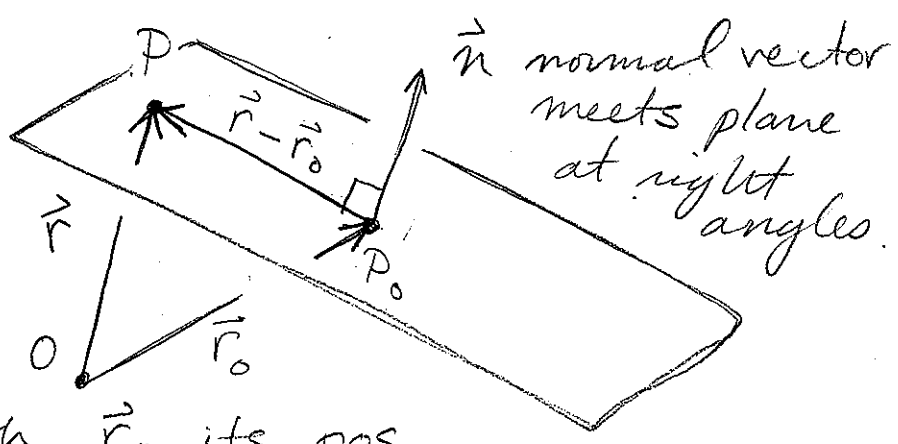
model has more parameters, and proj is onto a plane or similar. Cf. Math 415.]

Planes in \mathbb{R}^3 :

Take a point

$$P_0 = (x_0, y_0, z_0)$$

on the plane, with \vec{r}_0 its pos. vector.



Test if $P = (x, y, z)$ is in the plane:

\vec{n} and $\vec{r} - \vec{r}_0$ are perpendicular (orthogonal),

i.e. $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$

If $\vec{n} = (a, b, c)$, have $P = (x, y, z)$ in the plane when

$$0 = (a, b, c) \cdot (x - x_0, y - y_0, z - z_0)$$
$$= a(x - x_0) + b(y - y_0) + c(z - z_0)$$

$$= ax + by + cz + d$$

$$\text{with } d = -(ax_0 + by_0 + cz_0)$$

Conversely,

$$ax + by + cz + d = 0$$

defines a plane [unless $a = b = c = 0$.]

[Normal vectors will be a key concept later in the course. Can be used to solve geometric problems about planes]

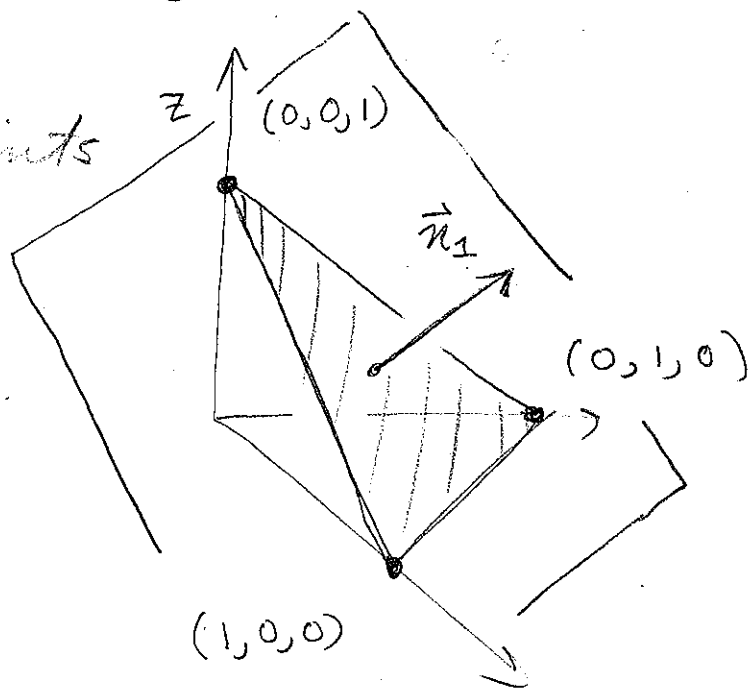
Ex: $P_1 =$ plane given by $x + y + z - 1 = 0$

(11)

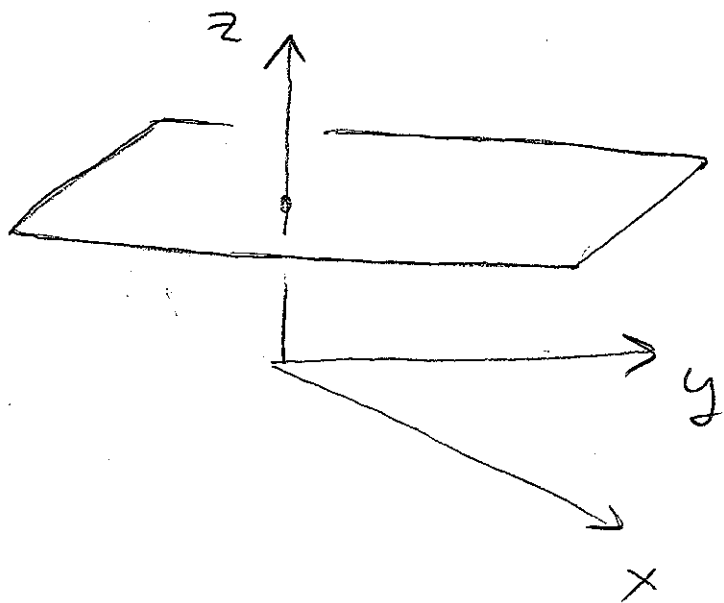
Plane is det by 3-points

Normal vector

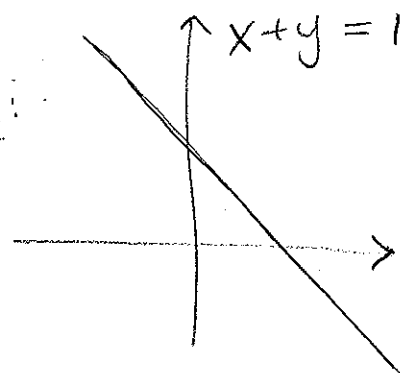
$$\vec{n}_1 = (1, 1, 1)$$



$$P_2 = \{ z = 1 \}$$



Compare:



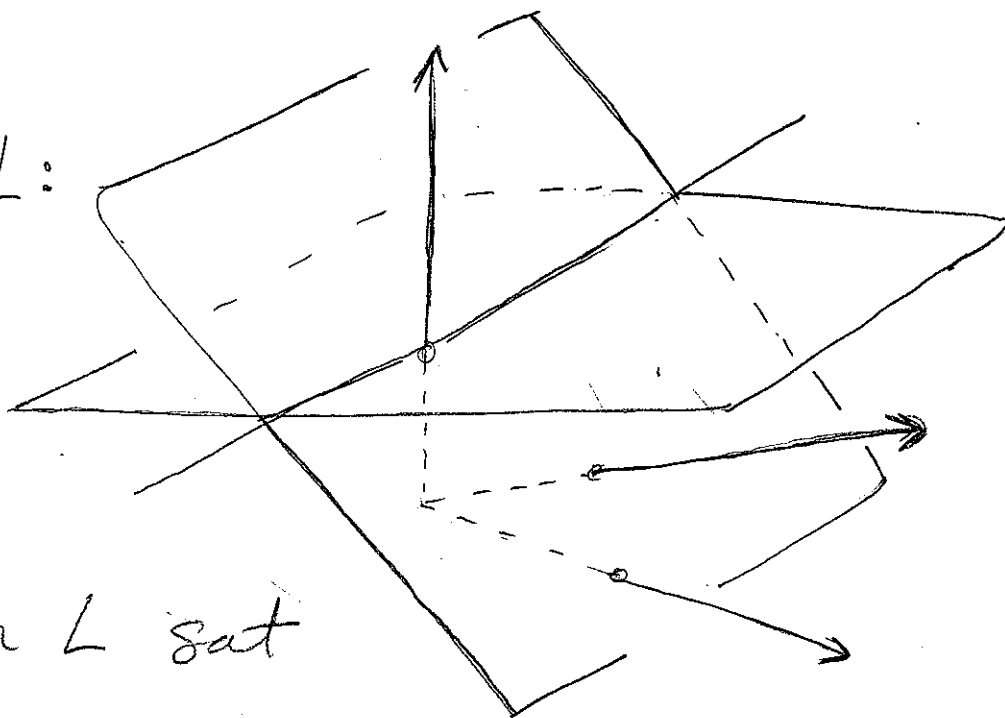
$$\vec{n}_2 = (0, 0, 1)$$

Q1: What is the intersection of P_1 and P_2 ?

Q2: What is the angle between P_1 and P_2 ?

A1:

A line L :



points on L sat

$$x + y + z = 1 \quad \text{and} \quad z = 1$$

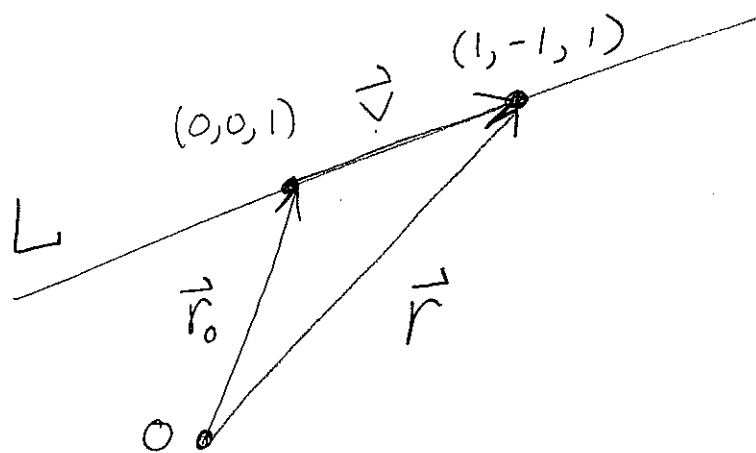
Two easy solutions:

$$\vec{r}_0 = (0, 0, 1)$$

$$\vec{r} = (1, -1, 1)$$

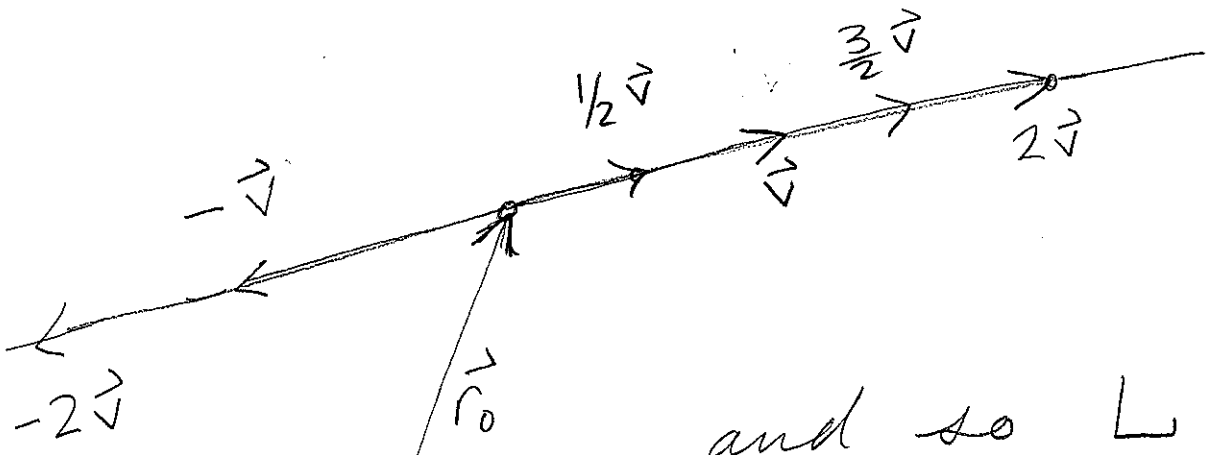
$$\vec{v} = \vec{r} - \vec{r}_0$$

$$= (1, -1, 0)$$



Every pt on L has the form

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$



and so L

is parameterized

by

$$(0, 0, 1) + t(1, -1, 0) = (t, -t, 1)$$

This is just like worksheet 3(e) from yesterday

