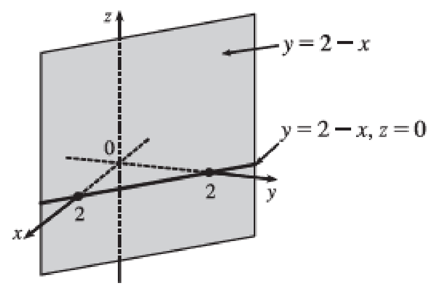
Section 12.1

1. We start at the origin, which has coordinates $(0, 0, 0)$. First we move 4 units along the positive x-axis, affecting only the x-coordinate, bringing us to the point $(4, 0, 0)$. We then move 3 units straight downward, in the negative z-direction. Thus only the z-coordinate is affected, and we arrive at $(4, 0, -3)$.

3. The distance from a point to the xz-plane is the absolute value of the y-coordinate of the point. $Q(-5, -1, 4)$ has the y-coordinate with the smallest absolute value, so Q is the point closest to the xz-plane. $R(0, 3, 8)$ must lie in the yz-plane since the distance from R to the yz-plane, given by the x-coordinate of R, is 0.

5. The equation $x + y = 2$ represents the set of all points in $\mathbb{R}^3$ whose x- and y-coordinates have a sum of 2, or equivalently where $y = 2 - x$. This is the set $\{(x, 2 - x, z) \mid x \in \mathbb{R}, z \in \mathbb{R}\}$ which is a vertical plane that intersects the xy-plane in the line $y = 2 - x$, $z = 0$.



11. An equation of the sphere with center $(1, -4, 3)$ and radius 5 is $(x - 1)^2 + [y - (-4)]^2 + (z - 3)^2 = 5^2$ or $(x - 1)^2 + (y + 4)^2 + (z - 3)^2 = 25$. The intersection of this sphere with the xz-plane is the set of points on the sphere whose y-coordinate is 0. Putting $y = 0$ into the equation, we have $(x - 1)^2 + 4^2 + (z - 3)^2 = 25$, $y = 0$ or $(x - 1)^2 + (z - 3)^2 = 9$, $y = 0$, which represents a circle in the xz-plane with center $(1, 0, 3)$ and radius 3.

25. The inequality $x > 3$ represents a half-space consisting of all points in front of the plane $x = 3$.

31. Here $x^2 + z^2 \le 9$ or equivalently $\sqrt{x^2 + z^2} \le 3$ which describes the set of all points in $\mathbb{R}^3$ whose distance from the y-axis is at most 3. Thus, the inequality represents the region consisting of all points on or inside a circular cylinder of radius 3 with axis the y-axis.

Section 12.2

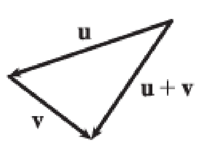
4. (a) The initial point of $\overrightarrow{QR}$ is positioned at the terminal point of $\overrightarrow{PQ}$, so by the Triangle Law the sum $\overrightarrow{PQ} + \overrightarrow{QR}$ is the vector with initial point P and terminal point R, namely $\overrightarrow{PR}$.

   (b) By the Triangle Law, $\overrightarrow{RP} + \overrightarrow{PS}$ is the vector with initial point R and terminal point S, namely $\overrightarrow{RS}$.

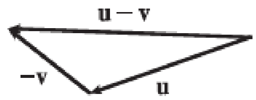
   (c) First we consider $\overrightarrow{QS} - \overrightarrow{PS}$ as $\overrightarrow{QS} + \left(-\overrightarrow{PS}\right)$. Then since $-\overrightarrow{PS}$ has the same length as $\overrightarrow{PS}$ but points in the opposite direction, we have $-\overrightarrow{PS} = \overrightarrow{SP}$ and so $\overrightarrow{QS} - \overrightarrow{PS} = \overrightarrow{QS} + \overrightarrow{SP} = \overrightarrow{QP}$.

   (d) We use the Triangle Law twice: $\overrightarrow{RS} + \overrightarrow{SP} + \overrightarrow{PQ} = \left(\overrightarrow{RS} + \overrightarrow{SP}\right) + \overrightarrow{PQ} = \overrightarrow{RP} + \overrightarrow{PQ} = \overrightarrow{RQ}$

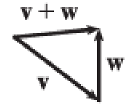
5. (a)



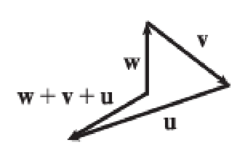
(b)



(c)



(d)



1. (a) $\mathbf{a} \cdot \mathbf{b}$ is a scalar, and the dot product is defined only for vectors, so $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$ has no meaning.

   (b) $(\mathbf{a} \cdot \mathbf{b})\,\mathbf{c}$ is a scalar multiple of a vector, so it does have meaning.

   (c) Both $|\mathbf{a}|$ and $\mathbf{b} \cdot \mathbf{c}$ are scalars, so $|\mathbf{a}|\,(\mathbf{b} \cdot \mathbf{c})$ is an ordinary product of real numbers, and has meaning.

   (d) Both $\mathbf{a}$ and $\mathbf{b} + \mathbf{c}$ are vectors, so the dot product $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$ has meaning.

   (e) $\mathbf{a} \cdot \mathbf{b}$ is a scalar, but $\mathbf{c}$ is a vector, and so the two quantities cannot be added and $\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$ has no meaning.

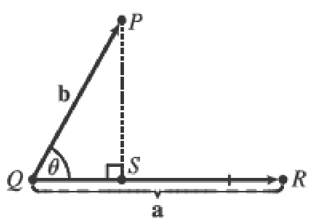
   (f) $|\mathbf{a}|$ is a scalar, and the dot product is defined only for vectors, so $|\mathbf{a}| \cdot (\mathbf{b} + \mathbf{c})$ has no meaning.

11. $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are all unit vectors, so the triangle is an equilateral triangle. Thus the angle between $\mathbf{u}$ and $\mathbf{v}$ is $60°$ and $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}| \cos 60° = (1)(1)\left(\frac{1}{2}\right) = \frac{1}{2}$. If $\mathbf{w}$ is moved so it has the same initial point as $\mathbf{u}$, we can see that the angle between them is $120°$ and we have $\mathbf{u} \cdot \mathbf{w} = |\mathbf{u}|\,|\mathbf{w}| \cos 120° = (1)(1)\left(-\frac{1}{2}\right) = -\frac{1}{2}$.

53. Consider the H—C—H combination consisting of the sole carbon atom and the two hydrogen atoms that are at $(1, 0, 0)$ and $(0, 1, 0)$ (or any H—C—H combination, for that matter). Vector representations of the line segments emanating from the carbon atom and extending to these two hydrogen atoms are $\langle 1 - \frac{1}{2}, 0 - \frac{1}{2}, 0 - \frac{1}{2} \rangle = \langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle$ and $\langle 0 - \frac{1}{2}, 1 - \frac{1}{2}, 0 - \frac{1}{2} \rangle = \langle -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \rangle$. The bond angle, θ, is therefore given by

$$\cos\theta = \frac{\langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle \cdot \langle -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \rangle}{\left|\langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle\right| \left|\langle -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \rangle\right|} = \frac{-\frac{1}{4} - \frac{1}{4} + \frac{1}{4}}{\sqrt{\frac{3}{4}}\sqrt{\frac{3}{4}}} = -\frac{1}{3} \quad \Rightarrow \quad \theta = \cos^{-1}\left(-\frac{1}{3}\right) \approx 109.5°.$$

43. (a)



The distance between a point and a line is the length of the perpendicular from the point to the line, here $\left|\overrightarrow{PS}\right| = d$. But referring to triangle PQS, $d = \left|\overrightarrow{PS}\right| = \left|\overrightarrow{QP}\right| \sin\theta = |\mathbf{b}| \sin\theta$. But θ is the angle between $\overrightarrow{QP} = \mathbf{b}$ and $\overrightarrow{QR} = \mathbf{a}$. Thus by Theorem 6, $\sin\theta = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|\,|\mathbf{b}|}$ and so $d = |\mathbf{b}| \sin\theta = \dfrac{|\mathbf{b}|\,|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$.

   (b) $\mathbf{a} = \overrightarrow{QR} = \langle -1, -2, -1 \rangle$ and $\mathbf{b} = \overrightarrow{QP} = \langle 1, -5, -7 \rangle$. Then

   $\mathbf{a} \times \mathbf{b} = \langle (-2)(-7) - (-1)(-5), (-1)(1) - (-1)(-7), (-1)(-5) - (-2)(1) \rangle = \langle 9, -8, 7 \rangle$.

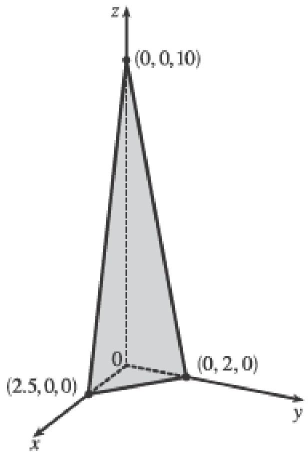
   Thus the distance is $d = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|} = \frac{1}{\sqrt{6}}\sqrt{81 + 64 + 49} = \sqrt{\frac{194}{6}} = \sqrt{\frac{97}{3}}$.

49. (a) No. If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = 0$, so $\mathbf{a}$ is perpendicular to $\mathbf{b} - \mathbf{c}$, which can happen if $\mathbf{b} \neq \mathbf{c}$. For example, let $\mathbf{a} = \langle 1, 1, 1 \rangle$, $\mathbf{b} = \langle 1, 0, 0 \rangle$ and $\mathbf{c} = \langle 0, 1, 0 \rangle$.

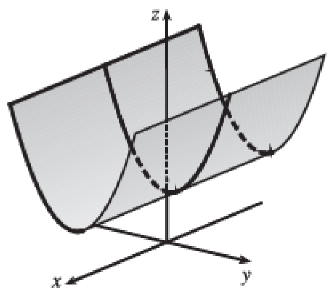
   (b) No. If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ then $\mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{0}$, which implies that $\mathbf{a}$ is parallel to $\mathbf{b} - \mathbf{c}$, which of course can happen if $\mathbf{b} \neq \mathbf{c}$.

   (c) Yes. Since $\mathbf{a} \cdot \mathbf{c} = \mathbf{a} \cdot \mathbf{b}$, $\mathbf{a}$ is perpendicular to $\mathbf{b} - \mathbf{c}$, by part (a). From part (b), $\mathbf{a}$ is also parallel to $\mathbf{b} - \mathbf{c}$. Thus since $\mathbf{a} \neq \mathbf{0}$ but is both parallel and perpendicular to $\mathbf{b} - \mathbf{c}$, we have $\mathbf{b} - \mathbf{c} = \mathbf{0}$, so $\mathbf{b} = \mathbf{c}$.

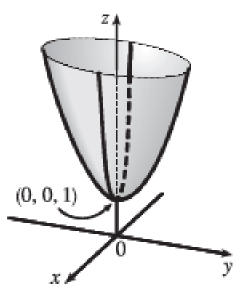
23. $z = 10 - 4x - 5y$ or $4x + 5y + z = 10$, a plane with intercepts 2.5, 2, and 10.



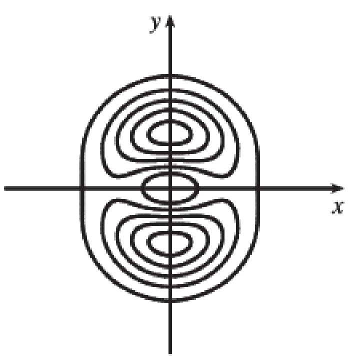
25. $z = y^2 + 1$, a parabolic cylinder



27. $z = 4x^2 + y^2 + 1$, an elliptic paraboloid with vertex at $(0, 0, 1)$.



34.



61. $k = x + 3y + 5z$ is a family of parallel planes with normal vector $\langle 1, 3, 5 \rangle$.

64. $k = x^2 - y^2$ is a family of hyperbolic cylinders oriented vertically. The cross section of each level surface in the xy-plane is a hyperbola with axis the x-axis when $k > 0$ and y-axis when $k < 0$. (When $k = 0$, the level surface is two intersecting vertical planes.)

Section 14.3

59. $u = \ln\sqrt{x^2+y^2} = \ln(x^2+y^2)^{1/2} = \frac{1}{2}\ln(x^2+y^2) \quad \Rightarrow \quad u_x = \dfrac{1}{2}\dfrac{1}{x^2+y^2}\cdot 2x = \dfrac{x}{x^2+y^2}$,

$u_{xy} = x(-1)(x^2+y^2)^{-2}(2y) = -\dfrac{2xy}{(x^2+y^2)^2}$ and $u_y = \dfrac{1}{2}\dfrac{1}{x^2+y^2}\cdot 2y = \dfrac{y}{x^2+y^2}$,

$u_{yx} = y(-1)(x^2+y^2)^{-2}(2x) = -\dfrac{2xy}{(x^2+y^2)^2}$. Thus $u_{xy} = u_{yx}$.

94. $f_x(0,0) = \lim\limits_{h\to 0}\dfrac{f(0+h,0)-f(0,0)}{h} = \lim\limits_{h\to 0}\dfrac{(h^3+0)^{1/3}-0}{h} = \lim\limits_{h\to 0}\dfrac{h}{h} = 1$.

Or: Let $g(x) = f(x,0) = \sqrt[3]{x^3+0} = x$. Then $g'(x) = 1$ and $g'(0) = 1$ so, by (1), $f_x(0,0) = g'(0) = 1$.

13. $f(x,y) = \dfrac{x}{x+y}$. The partial derivatives are $f_x(x,y) = \dfrac{1(x+y)-x(1)}{(x+y)^2} = y/(x+y)^2$ and $f_y(x,y) = x(-1)(x+y)^{-2}\cdot 1 = -x/(x+y)^2$, so $f_x(2,1) = \frac{1}{9}$ and $f_y(2,1) = -\frac{2}{9}$. Both f_x and f_y are continuous functions for $y \neq -x$, so f is differentiable at $(2,1)$ by Theorem 8. The linearization of f at $(2,1)$ is given by $L(x,y) = f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1) = \frac{2}{3} + \frac{1}{9}(x-2) - \frac{2}{9}(y-1) = \frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}$.

Section 14.5

55. (a) Since f is a polynomial, it has continuous second-order partial derivatives, and

$f(tx,ty) = (tx)^2(ty) + 2(tx)(ty)^2 + 5(ty)^3 = t^3x^2y + 2t^3xy^2 + 5t^3y^3 = t^3(x^2y + 2xy^2 + 5y^3) = t^3 f(x,y)$.

Thus, f is homogeneous of degree 3.

(b) Differentiating both sides of $f(tx,ty) = t^n f(x,y)$ with respect to t using the Chain Rule, we get

$$\frac{\partial}{\partial t} f(tx,ty) = \frac{\partial}{\partial t}[t^n f(x,y)] \quad \Leftrightarrow$$

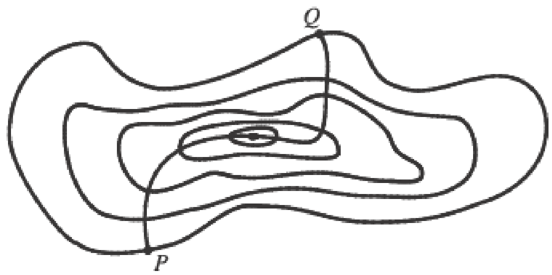
$$\frac{\partial}{\partial(tx)} f(tx,ty)\cdot\frac{\partial(tx)}{\partial t} + \frac{\partial}{\partial(ty)} f(tx,ty)\cdot\frac{\partial(ty)}{\partial t} = x\frac{\partial}{\partial(tx)} f(tx,ty) + y\frac{\partial}{\partial(ty)} f(tx,ty) = nt^{n-1} f(x,y).$$

Setting $t = 1$: $x\dfrac{\partial}{\partial x} f(x,y) + y\dfrac{\partial}{\partial y} f(x,y) = nf(x,y)$.

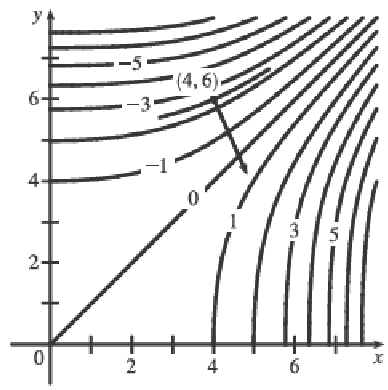
Section 14.6

1. We can approximate the directional derivative of the pressure function at K in the direction of S by the average rate of change of pressure between the points where the red line intersects the contour lines closest to K (extend the red line slightly at the left). In the direction of S, the pressure changes from 1000 millibars to 996 millibars and we estimate the distance between these two points to be approximately 50 km (using the fact that the distance from K to S is 300 km). Then the rate of change of pressure in the direction given is approximately $\frac{996-1000}{50} = -0.08$ millibar/km.

36. The curve of steepest ascent is perpendicular to all of the contour lines.



38. If we place the initial point of the gradient vector $\nabla f(4, 6)$ at $(4, 6)$, the vector is perpendicular to the level curve of f that includes $(4, 6)$, so we sketch a portion of the level curve through $(4, 6)$ (using the nearby level curves as a guideline) and draw a line perpendicular to the curve at $(4, 6)$. The gradient vector is parallel to this line, pointing in the direction of increasing function values, and with length equal to the maximum value of the directional derivative of f at $(4, 6)$. We can estimate this length by finding the average rate of change in the direction of the gradient. The line intersects the contour lines corresponding to -2 and -3 with an estimated distance of 0.5 units. Thus the rate of change is approximately $\dfrac{-2-(-3)}{0.5} = 2$, and we sketch the gradient vector with length 2.



Rest of Chapter 14 questions later, after Chapter 13 stuff.