Math 530: Problem Set 6

Due date: In class on Friday, March 20.
Course Web Page: `http://dunfield.info/530`
Warning: This assignment is longer than it appears; the first problem has not less than 9 parts.

1. Marcus, Chapter 4, Problem 17.

2. Marcus, Chapter 4, Problem 27.

3. Let K be a number field, $t > 0$ in $\mathbb{R}$. Show that the convex, centrally symmetric set

$$X = \left\{ (z_\tau) \in K_\mathbb{R} \;\middle|\; \sum_\tau |z_\tau| < t \right\}$$

   has (canonical) volume $2^r \pi^s t^n / n!$. Hint: One approach is to induct on r and s.

4. Let $\mathfrak{a}$ be an ideal of $\mathcal{O}_K$. Using only theorems stated in class, prove there exists an $a \neq 0$ in $\mathfrak{a}$ such that

$$\left| \mathcal{N}_{K/\mathbb{Q}}(a) \right| \leq M \mathcal{N}(\mathfrak{a}) \quad \text{where } M = \tfrac{n!}{n^n} \left(\tfrac{4}{\pi}\right)^s \sqrt{|\Delta_K|}.$$

   This is the Minkowski Bound, infamous on the 530 comp exams.

   Hint: Use problem 3 and the inequality between the arithmetic and geometric means:

$$\frac{1}{n} \sum_\tau |z_\tau| \geq \left(\prod_\tau |z_\tau| \right)^{1/n}$$