

Math 530: Problem Set 5

Due date: In class on Wednesday, March 4.

Course Web Page: <http://dunfield.info/530>

1. Marcus, Chapter 4, Problem 5, parts (a) and (b).
2. Marcus, Chapter 4, Problem 6.
3. Marcus, Chapter 4, Problem 18.
4. Marcus, Chapter 4, Problem 21.
5. Let p be an odd prime, and $\left(\frac{a}{p}\right)$ denote the Legendre symbol. Use elementary facts about \mathbb{F}_p^\times to prove that

$$\left(\frac{-1}{p}\right) = (-1)^{\frac{p-1}{2}} = \begin{cases} 1 & \text{if } p \equiv 1 \pmod{4} \\ -1 & \text{if } p \equiv 3 \pmod{4} \end{cases}$$

6. Let p be an odd rational prime, and consider $\mathbb{Q}(\zeta)$ where ζ is a primitive p^{th} -root of unity. Let $p^* = (-1)^{\frac{p-1}{2}} p$. Prove that $\mathbb{Q}(\sqrt{p^*}) \subset \mathbb{Q}(\zeta)$. Hint: consider

$$\tau = \sum_{a \in \mathbb{F}_p^\times} \left(\frac{a}{p}\right) \zeta^a \quad \text{and show } \tau^2 = p^*.$$

Midterm info

- **When:** The midterm will be Monday, March 9. This will be a two hour exam, which you can take either from 8:00-10:00 or 8:50-10:50 at your individual choice. It will be held in our usual classroom.
- **What:** The exam will consist of problems similar to, but easier than, those on the HW. In particular, I will not expect you to repeat proofs given in class, nor will any of the questions be identical to those on the HW. The problems may well resemble comp questions, but that's incidental from my point of view and not goal.
- **Cheat Sheet:** While the test will be closed book and notes, you will be allowed to bring in one sheet of letter or A4 paper on to which you can write/print/photocopy whatever you want (both sides permitted). The only limit is that you have to be able to read it without a magnifying device. In addition, calculators will not be permitted.
- **Office hours:** There will be extra office hours to help you prepare for the exam which will be announced shortly.
- **HW schedule:** There will be no HW due Wednesday, March 11. The next assignment will be due Friday, March 20.