Math 530: Problem Set 2

Due date: In class on Wednesday, February 11.[1]
Course Web Page: `http://dunfield.info/530`
Office hours: Monday 10-11, Tuesday 2:30 - 3:30, and by appointment. My office is Altgeld 378 (in the south wing).

1. Marcus, Chapter 2, Problem 22.

2. Marcus, Chapter 2, Problem 27.

3. Marcus, Chapter 2, Problem 29.

4. Marcus, Chapter 2, Problem 30.

5. Marcus, Chapter 3, Problem 4.

6. Complete the proof that Dedekind domains have unique factorization of ideals by showing that any two such factorizations are the same up to reordering. Do this using the context and lemmas of the lecture on February 4, rather than those from the text.

7. Show that the quotient ring $\mathcal{O}/\mathfrak{a}$ of a Dedekind domain by an ideal $\mathfrak{a} \neq 0$ is a PID.

   Hint: Use the Chinese Remainder Theorem to reduce to the case when $\mathfrak{a} = \mathfrak{p}^n$ where $\mathfrak{p}$ is prime. Prove that the only proper ideals of $\mathcal{O}/\mathfrak{p}^n$ are given by $\mathfrak{p}^k/\mathfrak{p}^n$ where $1 \leq k \leq n-1$. Then show each of these is principal.

8. Use the preceding problem to show that every ideal of a Dedekind domain is generated by two elements.

[1] Revised version of April 8, 2009, removing problem 9.