HW 8 SOLUTIONS, MA525

Hatcher 2.1 Problem 11

Let $r : X \to A$ be the retraction and let $i : A \to X$ be the inclusion. Then $r \circ i$ is the identity map on A. If r_* and i_* are maps induced at the level of homology, then $r_* \circ i_*$ has to be the identity map on $H_*(A)$. This implies that i_* has to be injective.

Hatcher 2.1 Problem 15

If $C = 0$, then $Ker(B \to C) = B$. Since the sequence is exact, $Im(A \to B) = Ker(B \to C) = B$ i.e. the map is surjective. Similarly, $Im(C \to D) = 0$ and by exactness, $Ker(D \to E) = Im(C \to D) = 0$ i.e the map is injective.

Conversely, suppose that $A \to B$ is surjective and $D \to E$ is injective. Exactness implies: $Ker(B \to C) = B$ so $Im(B \to C) = 0$, which equals $Ker(C \to D)$, so the map $C \to D$ is injective. But $Im(C \to D) = Ker(D \to E) = 0$ since $D \to E$ is injective. Thus $C \to D$ is an injective map with image 0. This means $C = 0$.