Math 525: Problem Set 2[1]

Due date: In class on Wednesday, September 9.
Course Web Page: `http://dunfield.info/525`
Office hours: Mondays from 11-12, Tuesdays from 11:15 - 12:15, and by appointment. For an appointment, just talk to me after class, or email me at `nmd@illinois.edu`.
Required Text: Allen Hatcher, *Algebraic Topology*,
`http://www.math.cornell.edu/~hatcher/AT/ATpage.html`

1. Recall that a topological space X is *connected* if it is not the *disjoint* union of two non-empty open sets. The space X is *path-connected* if every pair of points can be joined by a path.

   (a) Prove directly from the least upper bound property that $\mathbb{R}$ is connected.

   (b) Is every path-connected space is also connected? Prove your answer. (In class, an example was give of a connected space that's not path-connected)

2. A map $p\colon \widetilde{X} \to X$ is a local homeomorphism if for every $\tilde{x} \in \widetilde{X}$ has an open neighborhood U so that $p|_U$ is a homeomorphism.

   (a) Prove that if $\widetilde{X}$ is compact and Hausdorff[2], then p is a covering map. (While every covering map is a local homeomorphism, the converse isn't always true as we saw in class.)

   (b) Again assuming $\widetilde{X}$ is compact and Hausdorff, prove that for each $x_0 \in X$, the set $p^{-1}(x_0)$ is finite. If X is path connected, show that the number of points in $p^{-1}(x_0)$ is independent of the choice of x_0. The size of $p^{-1}(x_0)$ is called the *degree* of p.

3. Hatcher, Section 1.3, Problem 1.

4. Hatcher, Section 1.3, Problem 6.

N.B. The problems removed from this assignment will appear on the next problem set.

[1]Revised version of September 8.

[2]A topological space is Hausdorff if for every two points x and y there are *disjoint* open sets U and V with $x \in U$ and $y \in V$. A metric space is always Hausdorff, and we will rarely consider spaces which don't have this property.