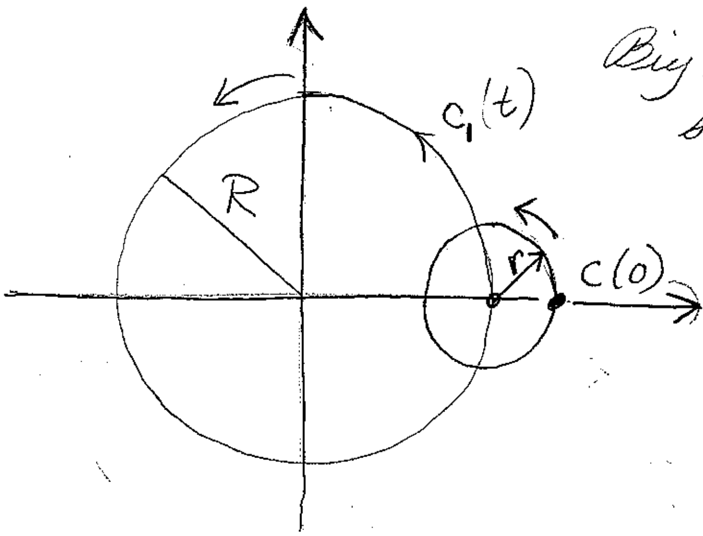
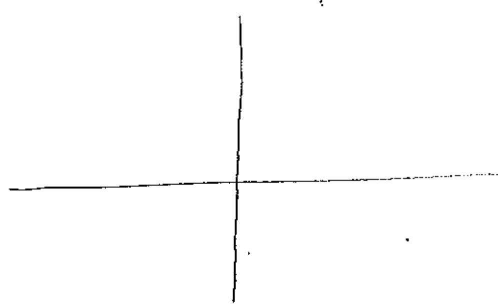
Lecture 29: Midterm review – exam tomorrow.

Note: curl not on the exam.

Parameterizations: Ch 3 Review Exercise # 12

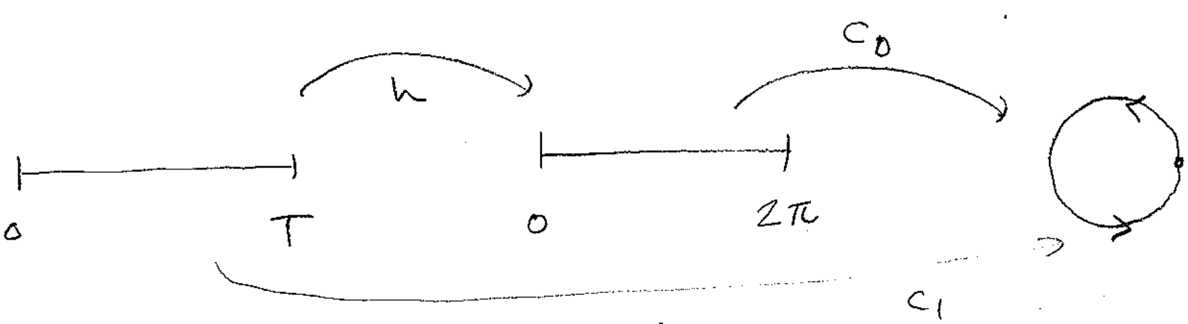


Big circle rotates in time T.
small in time T/4



Lets start with param the big circle $c_1 : [0,T] \to \mathbb{R}^2$

Start with some param $c_0 : [0, 2\pi] \to \mathbb{R}^2$ of big circle $c_0(t) = (R\cos t, R\sin t)$



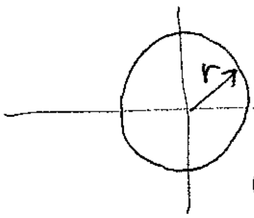
$h(0) = 0$

$h(T) = 2\pi$

$h' > 0$

$$h(t) = \left(\frac{2\pi}{T}\right)t$$

$$c_1(t) = \left(R\cos\left(\frac{2\pi}{T}\right)t,\ R\sin\left(\frac{2\pi}{T}\right)t \right)$$

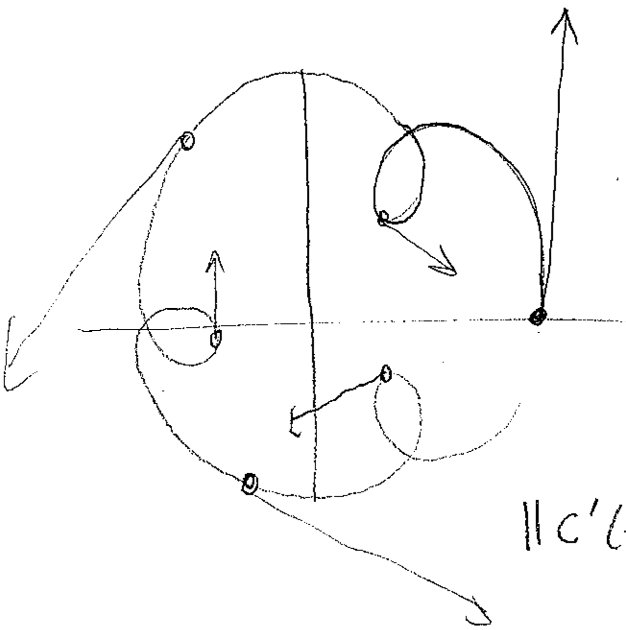


$$C_2 : [0,T] \longrightarrow \mathbb{R}^2$$

$$C_2(t) = \left(r\cos\left(\frac{8\pi}{T}\right)t \ , \ r\sin\left(\frac{8\pi}{T}\right)t \right)$$

$$C : [0,T] \longrightarrow \mathbb{R}^2$$

$$C(t) = C_1(t) + C_2(t) = \left(R\cos\frac{2\pi t}{T} + r\cos\frac{8\pi t}{T} \ , \ R\sin\frac{2\pi t}{T} + r\sin\frac{8\pi t}{T} \right)$$



Take $R = 2 \quad r = 1 \quad T = 2\pi$

$$c(t) = (2\cos t + \cos 4t, \ 2\sin t + \sin 4t)$$

$$c'(t) = (-2\sin t - 4\sin 4t, \ 2\cos t + 4\cos 4t)$$

$$\|c'(t)\| = \sqrt{20 + 16\cos 3t}$$

min speed: 2
max speed: 6

$$\text{Length} = \int_0^{2\pi} \|c'(t)\| \, dt \approx 26.72$$

$$f(x,y) = x^2 \qquad \int_C f \, ds = \int_0^{2\pi} f(c(t)) \, \|c'(t)\| \, dt$$

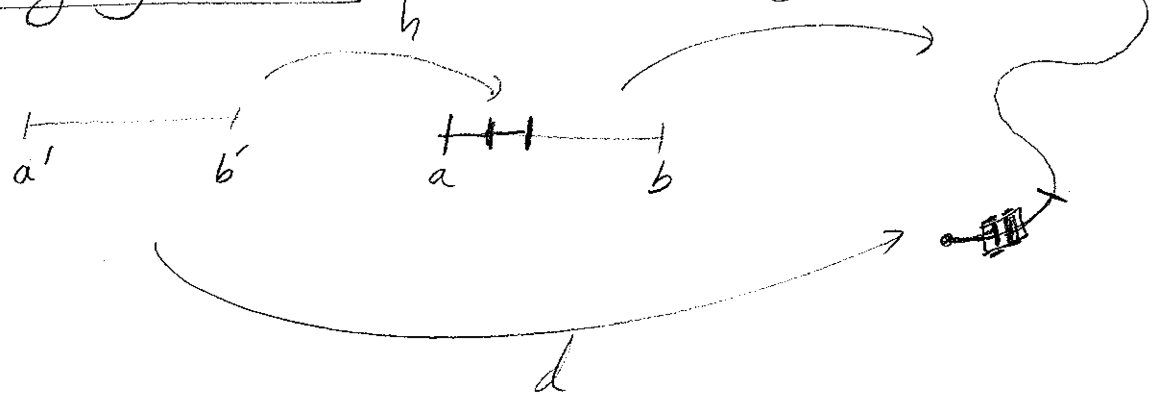
$$= \int_0^{2\pi} (2\cos t + \cos 4t)^2 \sqrt{20 + 16\cos 3t} \, dt \approx 78.98$$

$$\vec{F} = (y, x)$$

$$\int_C F \cdot ds = \int_0^{2\pi} F(c(t)) \cdot c'(t) \, dt = \int_0^{2\pi} \text{mess} \, dt$$

$= 0$ as $\vec{F}$ is conservative!

Changing speeds



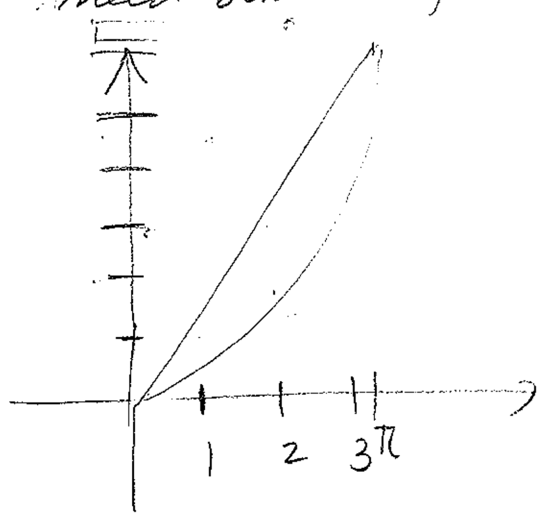
Suppose c is unit speed, e.g. $c(t) = (\cos t, \sin t)$

In this case the t param in $c(t)$ can equally be viewed as time or distance from the starting point.

h is the instructions of the form $h(t) =$ milepost to be at time t.

Suppose we want a param of the circle with non-const speed, need something with

say $h(0) = 0$
$h(1) = 1$
$h(2) = 3$
$h(\pi) = 2\pi$

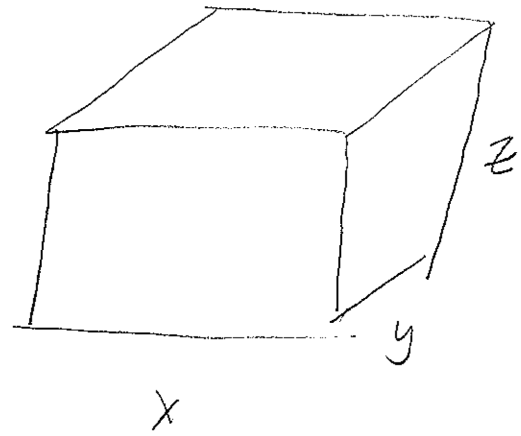


or really anything that is not a straight line.

Lagrange mult:

6 m^2 of material

max volume.



$V = xyz$

$A = 2xy + 2xz + 2yz = 6 \qquad x, y, z > 0$

$\nabla V = \lambda \nabla A$

$(yz, xz, xy) = \lambda 2 (y+z, x+z, x+y)$

$\Rightarrow \dfrac{yz}{y+z} = \dfrac{xz}{x+z} \Rightarrow (x+z)yz = (y+z)xz$

$yz^2 = xz^2 \Rightarrow x = y$

Also

$\dfrac{yz}{y+z} = \dfrac{xy}{x+y} \Rightarrow y^2 z = xy^2 \Rightarrow x = z$

So $x = y = z$ and $A = 6 \Rightarrow 6x^2 = 6 \Rightarrow x = 1$
$y = 1$
$z = 1$

So only one crit pt.

[Still need to deal with when x, y, z are small
to know there is a global max.]

Plane given by $\overbrace{ax+by+cz+d}^{g}=0$

$d \neq 0$

Minimize

$f(x,y,z)=x^2+y^2+z^2$ on $\uparrow$ given that such a minimum exists.

$$\nabla f = \lambda \nabla g = \lambda(a,b,c)$$

$$\| $$

$$(2x,2y,2z) \iff \begin{aligned} 2x &= \lambda a \\ 2y &= \lambda b \\ 2z &= \lambda c \end{aligned}$$

$$g=0 \iff a\left(\frac{\lambda}{2}a\right)+b\left(\frac{\lambda}{2}b\right)+c\left(\frac{\lambda}{2}c\right)+d=0$$

$$\Rightarrow \frac{\lambda}{2}(a^2+b^2+c^2)=-d \Rightarrow \cancel{a^2+b^2+c^2}$$

$$\lambda = \frac{-2d}{a^2+b^2c^2}$$

$$x = -\frac{ad}{a^2+b^2+c^2} \qquad y=\frac{-bd}{a^2+b^2+c^2}$$

$$z=\frac{-cd}{a^2+b^2+c^2}.$$